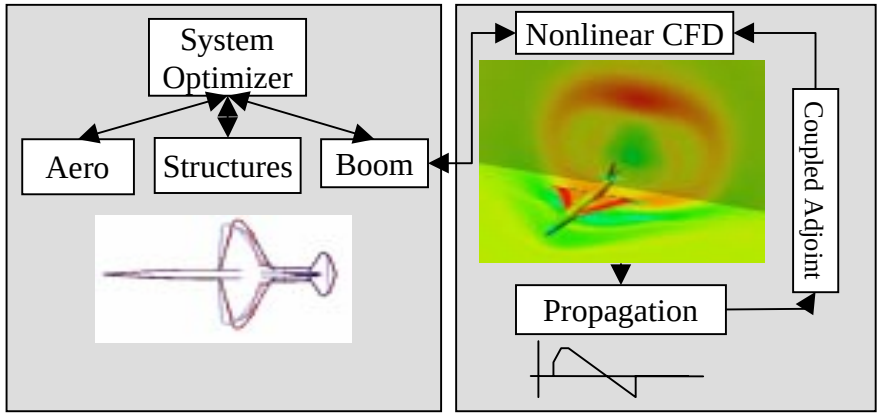
Aerodynamic Design Using The Euler and Navier-Stokes Equations



Multidisciplinary Optimization

High Fidelity Boom Analysis & Design

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Outline

- **Motivation & background**
- **Aerodynamic shape optimization problems**
- **Sensitivity / gradient computation for optimization**
  - *Adjoint-based sensitivity calculation*
  - *Complex-step formulation*
  - *Reduced-order models based on Proper Orthogonal Decomposition*
- **Sample results and other applications**
- **Conclusions**

Disclaimer

- ***Although Stanford University is heavily involved in ASCI as an Alliance member, this talk describes some of the work we are doing in our laboratory outside of ASCI***

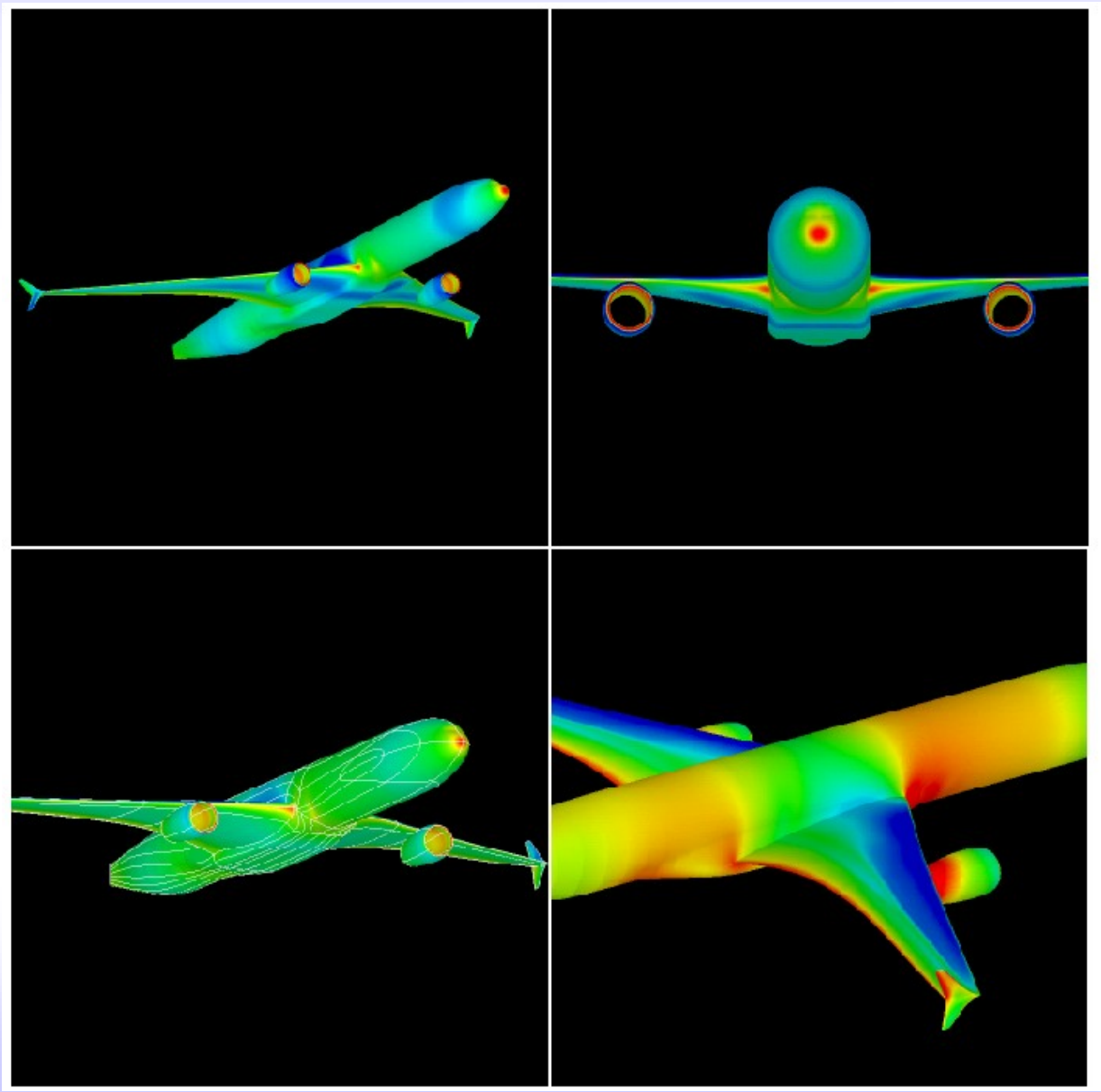
- ***Our ASCI project involves the prediction of the complex unsteady flow physics inside jet engines. Our work within the project involves:***
  - *Advanced algorithms for unsteady flow integration*
  - *Efficient nonlinear time-harmonic algorithms for unsteady periodic flow simulations*
  - *Advanced turbulence modeling*
  - *Integrated compressor-combustor-turbine simulations*
  - *Large-scale high-performance parallel implementations*
  - *Large-scale visualization using parallel rendering in clusters*

Motivation

- ***Our efforts at the Aerospace Computing Laboratory are aimed at developing the necessary tools to facilitate the multidisciplinary design of aerospace systems (aircraft, spacecraft, engines, distributed systems)***

- ***In particular, for aircraft that spend most of their time in the cruise condition, the design of aerodynamic shapes with optimum performance is an absolute necessity to meet requirements of:***
  - *Range*
  - *Cruise Mach number*
  - *Payload / Passengers*
  - *Takeoff / Landing field length*

Traditional Aircraft Configurations

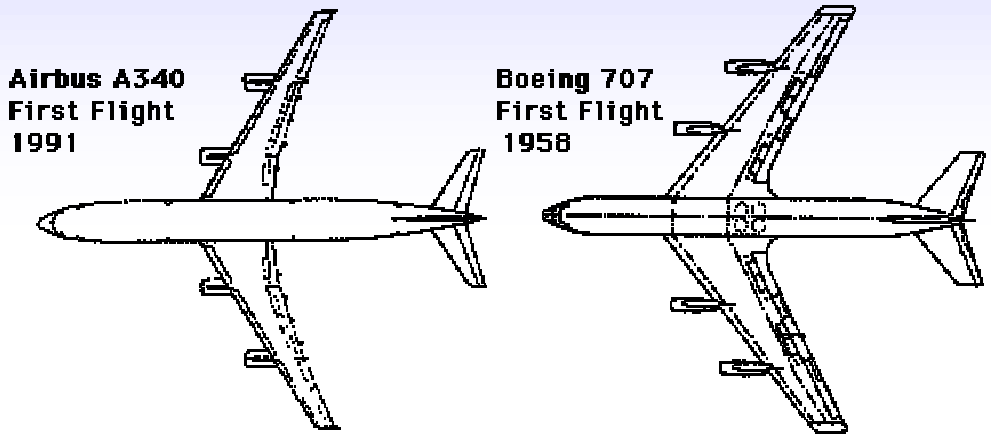


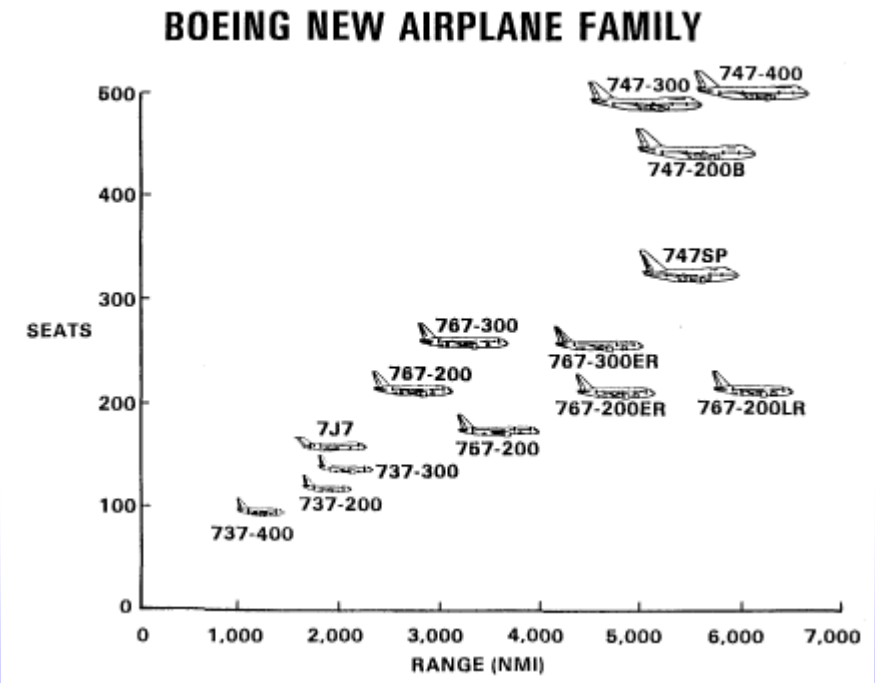
Traditional Design

- For traditional aircraft configurations where a large experience database has been accumulated over the years, design is evolutionary: a skillful designer can obtain close to optimal solutions through trial-and-error

- Although substantial improvements have been made under-the-hood, the fact remains that aerodynamic configurations for transonic commercial transports have remained virtually unchanged since the B-707

B707 vs. A340





- ***Seemingly identical aircraft***
- ***Improvements include***
  - *Active control*
  - *New Airfoil Concepts*
  - *Composite structures*
  - *Propulsion*
  - *MDO*
- ***Large economic risks introduce conservatism into new designs***

Revolutionary Design

- *Major advances in aircraft performance can still be obtained with a combination of revolutionary concepts that combine advanced technologies onto a given airframe*

- *The field of Multidisciplinary Design Optimization (MDO) where synergism between the participating disciplines in a design is exploited is necessary to obtain dramatic improvements in performance*

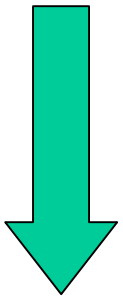
Revolutionary Designs

Revolutionary Design

- *This kind of aircraft has to be designed without the benefit of years of experience.*
- *What are the realistic chances that a designer may obtain an optimal design though trial-and-error? My guess is as good as yours….*
- *Automatic optimization techniques (analysis tools cleverly coupled to constrained nonlinear optimizers) become necessary*
- *How can we do this efficiently?*

Vision

- ***Effective simulation***



- ***Simulation-based design***

Automatic Optimization-Based Design

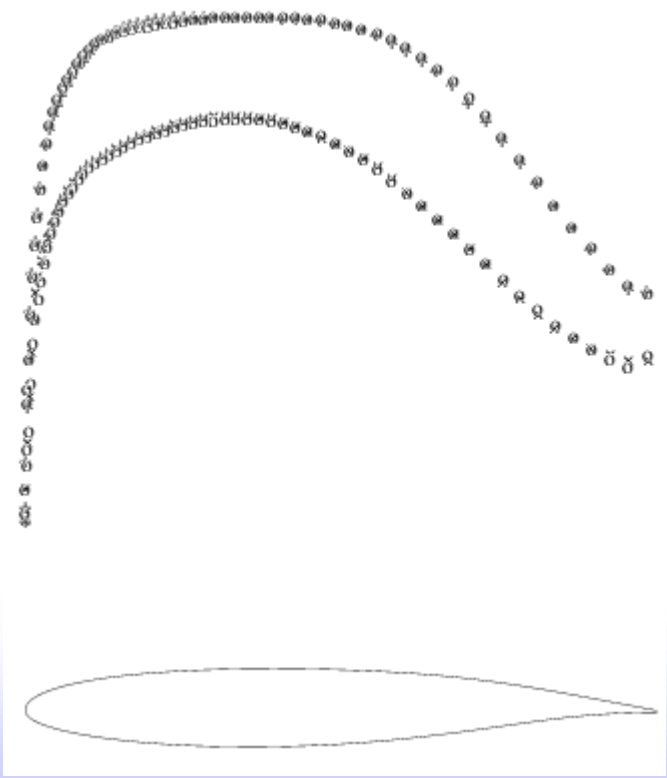
- ***The requirements to automate the design process are***
  - *Efficient, accurate, robust aerodynamic analysis*
  - *Low cost sensitivity analysis for large design spaces*
  - *Efficient nonlinear constrained optimization techniques*
- ***For aerodynamic design problems, the number of design variables is often very large (N ~ 500 - 1,000) and the computation of sensitivities of cost functions and constraints (needed for gradient-based optimizers) typically dominates the CPU time required***
- ***Must develop efficient algorithms for sensitivity calculation with large numbers of design variables***

Automatic Design Based on Control Theory

- One such method is the control theory-based adjoint method
- A wing is a device to generate lift (with minimum drag) by controlling the flow
- Apply theory of optimal control of systems governed by PDEs (Lions) with boundary control (the wing shape)
- Merge control theory and CFD

Design for Specified Pressure Distribution

- *Engineers know desirable characteristics of the surface pressure distribution, say, to prevent boundary layer separation, or delay transition to turbulence*
- *Specify target pressure*
- *However, not every target pressure is attainable (No body with stagnation pressure everywhere).*
- *An optimal control-based method should still work*



Control Theory Approach to the Design Problem

- *Define a cost function, for example*

$$I = \frac{1}{2}\int_{\mathcal{B}} (p - p_t)^2 \, d\mathcal{B}$$

- *or*

$$I = \frac{1}{2}\int_{\mathcal{B}} (q - q_t)^2 \, d\mathcal{B}$$

- *The surface shape is now treated as the control, which is to be varied to minimize* I, *subject to the constraint that the flow equations are satisfied in the domain* D

Automatic Shape Design Via Control Theory

- *Apply the theory of control of partial differential equations (of the flow) by boundary control (the shape)*
- *Find the Frechet derivative (infinite dimensional gradient) of the drag (or other performance measure) with respect to the shape by solving the adjoint equation in addition to the flow equation*
- *Modify the shape in the sense defined by the smoothed gradient*
- *Repeat the iterations until the performance value approaches an optimum*

Formulation of the Control Problem

Suppose that the surface of the body is expressed by an equation

$$f(\underline{x}) = 0$$

Vary f to $f + \delta f$ and find δI.

If we can express

$$\delta I = \int_{\mathcal{B}} g \delta f d\mathcal{B} = (g, \delta f)_{\mathcal{B}}$$

Then we can recognize g as the gradient $\frac{\partial I}{\partial f}$.

Choose a modification

$$\delta f = -\lambda g$$

Then to first order

$$\delta I = -\lambda \, (g, g)_{\mathcal{B}} < 0$$

- ***In the presence of constraints project g into the admissible trial space. Accelerate by use of smoothing and Quasi-Newton / Krylov subspace methods***

Traditional Approach to Optimization

The simplest approach to optimization is to define the geometry through a set of design parameters, which may, for example, be the weights α_i applied to a set of shape functions $b_i(x)$ so that the shape is represented as

$$f(x) = \sum \alpha_i b_i(x).$$

Then a cost function I is selected. The sensitivities $\frac{\partial I}{\partial \alpha_i}$ may now be estimated by making a small variation $\delta\alpha_i$ in each design parameter in turn and recalculating the flow to obtain the change in I. Then

$$\frac{\partial I}{\partial \alpha_i} \approx \frac{I(\alpha_i + \delta\alpha_i) - I(\alpha_i)}{\delta\alpha_i}.$$

The gradient vector $\frac{\partial I}{\partial \alpha}$ may now be used to determine a direction of improvement.

Traditional Approach to Optimization

The simplest procedure is to make a step in the negative gradient direction by setting

$$\alpha^{n+1} = \alpha^n - \lambda \delta\alpha,$$

so that to first order

$$I + \delta I = I - \frac{\partial I^T}{\partial \alpha}\delta\alpha = I - \lambda \frac{\partial I^T}{\partial \alpha}\frac{\partial I}{\partial \alpha}.$$

Adjoint-Based Design

The progress of the design procedure is measured in terms of a cost function

$$I = I\left(w, \mathcal{F}\right),$$

where w are the flow-field variables and $\mathcal{F}$ is the location of the boundary.

A change in $\mathcal{F}$ results in a change

$$\delta I = \left[\frac{\partial I^T}{\partial w}\right]_I \delta w + \left[\frac{\partial I^T}{\partial \mathcal{F}}\right]_{II} \delta \mathcal{F}, \tag{3}$$

in the cost function. [1]

[1] the subscripts I and II are used to distinguish the contributions due to the variation δw in the flow solution from the change associated directly with the modification $\delta \mathcal{F}$ in the shape

Adjoint-Based Design

The governing equations of the flow field

$$R\left(w, \mathcal{F}\right) = 0. \tag{4}$$

which express the dependence of w and $\mathcal{F}$ are introduced as a constraint within the flow-field domain $\mathcal{D}$. Then δw is determined from the equation

$$\delta R = \left[\frac{\partial R}{\partial w}\right]_{I} \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right]_{II} \delta \mathcal{F} = 0. \tag{5}$$

Next, introducing a Lagrange Multiplier ψ, we have

$$\begin{aligned} \delta I &= \frac{\partial I^T}{\partial w}\delta w + \frac{\partial I^T}{\partial \mathcal{F}}\delta \mathcal{F} - \psi^T \left(\left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F} \right) \\ &= \left\{ \frac{\partial I^T}{\partial w} - \psi^T \left[\frac{\partial R}{\partial w}\right] \right\}_{I} \delta w + \left\{ \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\}_{II} \delta \mathcal{F}. \end{aligned} \tag{6}$$

Adjoint-Based Design

Choosing ψ to satisfy the adjoint equation

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \frac{\partial I}{\partial w} \tag{7}$$

The first term is eliminated, and we find that

$$\delta I = \mathcal{G}\delta\mathcal{F}, \tag{8}$$

where

$$\mathcal{G} = \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right].$$

Once equation (8) is established, an improvement can be made with a shape change

$$\delta\mathcal{F} = -\lambda\mathcal{G}$$

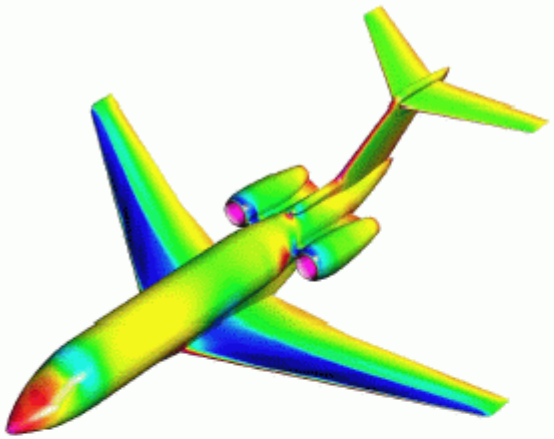
where λ is positive, and small enough that the first variation is an accurate estimate of δI. The variation in the cost function then becomes

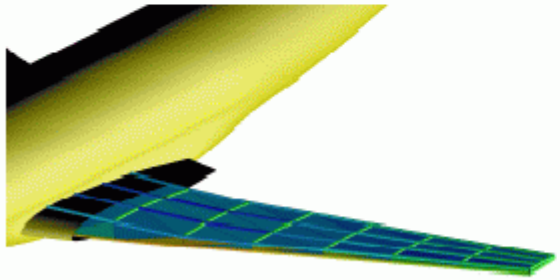
$$\delta I = -\lambda\mathcal{G}^T\mathcal{G} < 0.$$

Adjoint-Based Design

- **Main advantages:**
  - *The cost of a complete gradient evaluation, independently of the number of design variables, can be obtained with just one flow solution and one adjoint solution*
  - *Adjoint-based gradients require lower levels of convergence in the flow solver than the finite-difference method*
- **Disadvantages**
  - *An adjoint equation must be developed for each new governing equation set*
  - *A new boundary condition must be implemented for different cost functions*

Other Options For Sensitivity Analysis





- **Motivation:**
  - Gradient-based optimization requires accurate sensitivity information.
- **Typical approaches:**
  - **Finite-Difference:** easy to implement, but lacks robustness and accuracy.
  - **Algorithmic/Automatic/Computational Differentiation:** accurate, ease of implementation varies.
  - **Analytic Methods:** efficient and accurate, but long development and implementation times.
- **Complex-Step Method:** accurate and robust; easy to implement and maintain.

Finite-Difference Approximations

From Taylor series expansion.

- Forward-difference:

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} + \mathcal{O}(h)$$

- Central-difference:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} + \mathcal{O}(h^2)$$

Any finite-difference formula subject to subtractive cancellation.

Complex-Step Method

Can also be derived from a Taylor series expansion about x with a complex step ih:

$$f(x+ih) = f(x) + ihf'(x) - h^2\frac{f''(x)}{2!} - ih^3\frac{f'''(x)}{3!} + \dots$$

$$\Rightarrow f'(x) = \frac{\text{Im}\left[f(x+ih)\right]}{h} + h^2\frac{f'''(x)}{3!} + \dots$$

$$\Rightarrow \boxed{f'(x) \approx \frac{\text{Im}\left[f(x+ih)\right]}{h}}$$

No subtraction! Second order approximation.

Complex-Step Method

- *Although the cost is still linearly dependent on the number of design variables (with a proportionality constant), the gradient value is step insensitive*
- *Great advantage with respect to finite-difference methods in terms of robustness due to step insensitivity*
- *We have developed automatic procedures to "complexify" existing FORTRAN, C, and C++ codes*
- *Very useful for validation and debugging of more complex analytic methods*

Proper Orthogonal Decomposition

- There is a need for high-fidelity models in the multidisciplinary design and optimization of aerospace systems, but the computational cost is unfeasible for realistic problems.
- Response surfaces have disadvantages because the polynomial interpolate has no physical basis and there is no way to tell how well an approximate solution will agree with the exact solution.
- We are investigating POD as an alternate means to form approximate, reduced order models for use in the design environment.
- Initially we are investigating POD based models for use in Aerodynamic Shape Optimization problems but we are in the process of applying it to other problems where the method can be superior to response surfaces.

Proper Orthogonal Decomposition

- POD has its roots in statistical analysis and has appeared with various names, including: principal component analysis, empirical eigenfunctions, Karhunen-Loéve decomposition, and empirical orthogonal eigenfunctions.

- We are seeking representations of a function, $u(x)$, in terms of a basis $\{\varphi_j(x)\}_{j=1}^{\infty}$ which allows an approximation to be constructed as

$$u_M = \sum_{j=1}^{M} \eta_j \, \varphi_j(x) \tag{1}$$

- We would like to choose these basis functions such that they describe a typical function in the ensemble $\{\mathbf{u}^k\}$ better than any other linear basis.

Proper Orthogonal Decomposition

- The FLO82 flow solver, using a 5-stage Runge-Kutta scheme and multigrid, requires the equivalent of approximately *1,000* residual evaluations over the entire domain when artificial dissipation evaluations are also considered.

- The cost of the approximate flow solution from a set of basis modes is equivalent to about *50* evaluations of the residual over the entire domain.

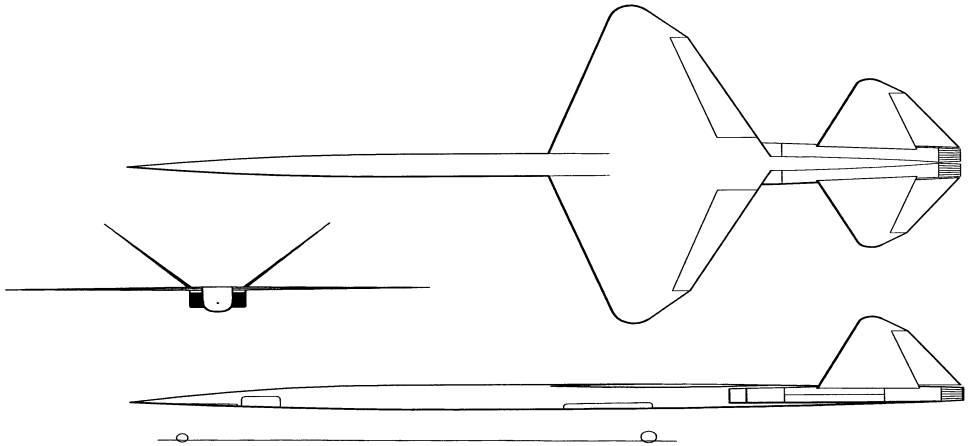
- Over an order of magnitude decrease in computational cost is achieved for two-dimensional flows, and the reduction in three-dimensions is even greater.

QSP Program Objectives

- ***To design a supersonic small aircraft with low enough sonic boom that it can be allowed to fly supersonically over land***

  - *Cruise Mach number = 2.4*
  - *TOGW ~ 100,000 lbs*
  - *Range = 6,000 nmi*
  - *Maximum overpressure = 0.3 psf*

NLF Concept – Tracy, Kroo

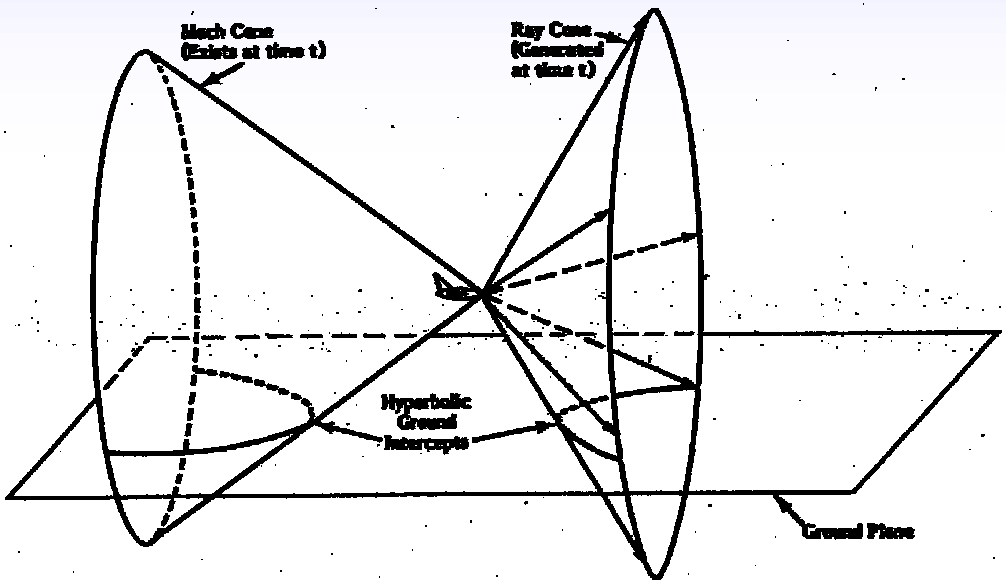


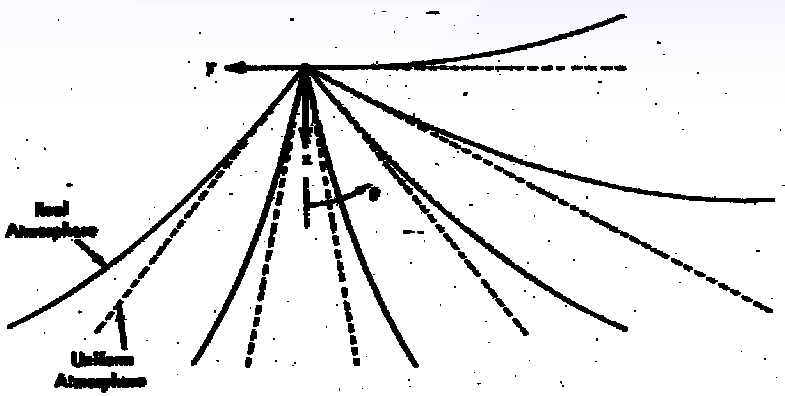
- ***Can this be achieved? Extremely difficult proposition; may lead to relaxed design requirements***

Conflicting Program Requirements

- ***Low boom vs. aerodynamic performance for range and payload***
- ***Low weight vs. range and payload requirements***
- ***High Mach number vs. boom and aero requirements***
- ***Overall, the most demanding requirement is the low boom requirement which has been somewhat arbitrarily set based on***
  - *Early 1990's HSR acceptability studies (NASA Langley)*
  - *Gulfstream marketing analysis*

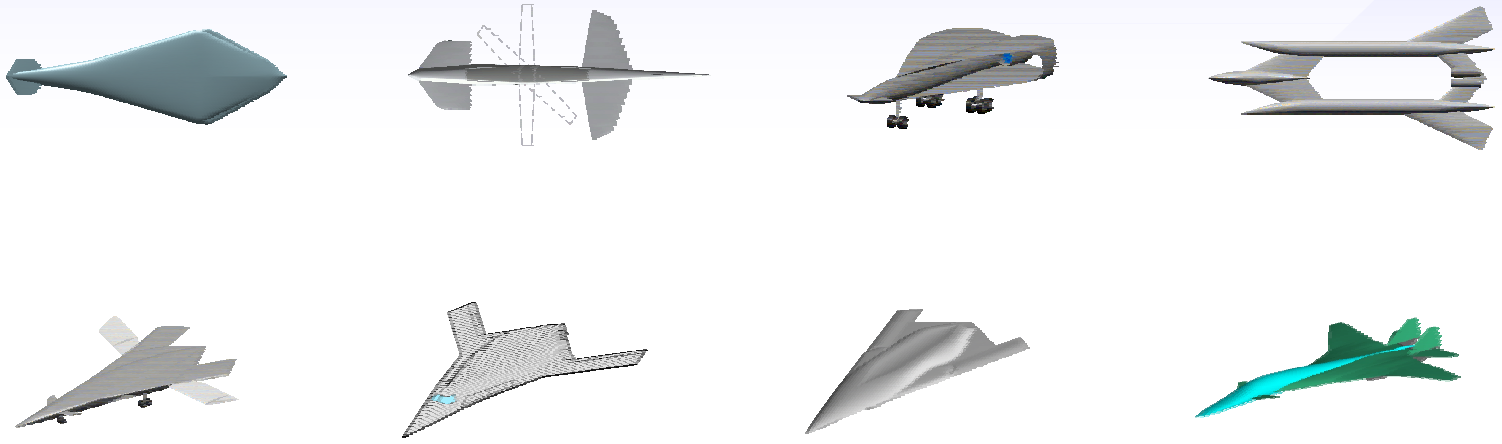
Fundamentals of Sonic Boom Theory





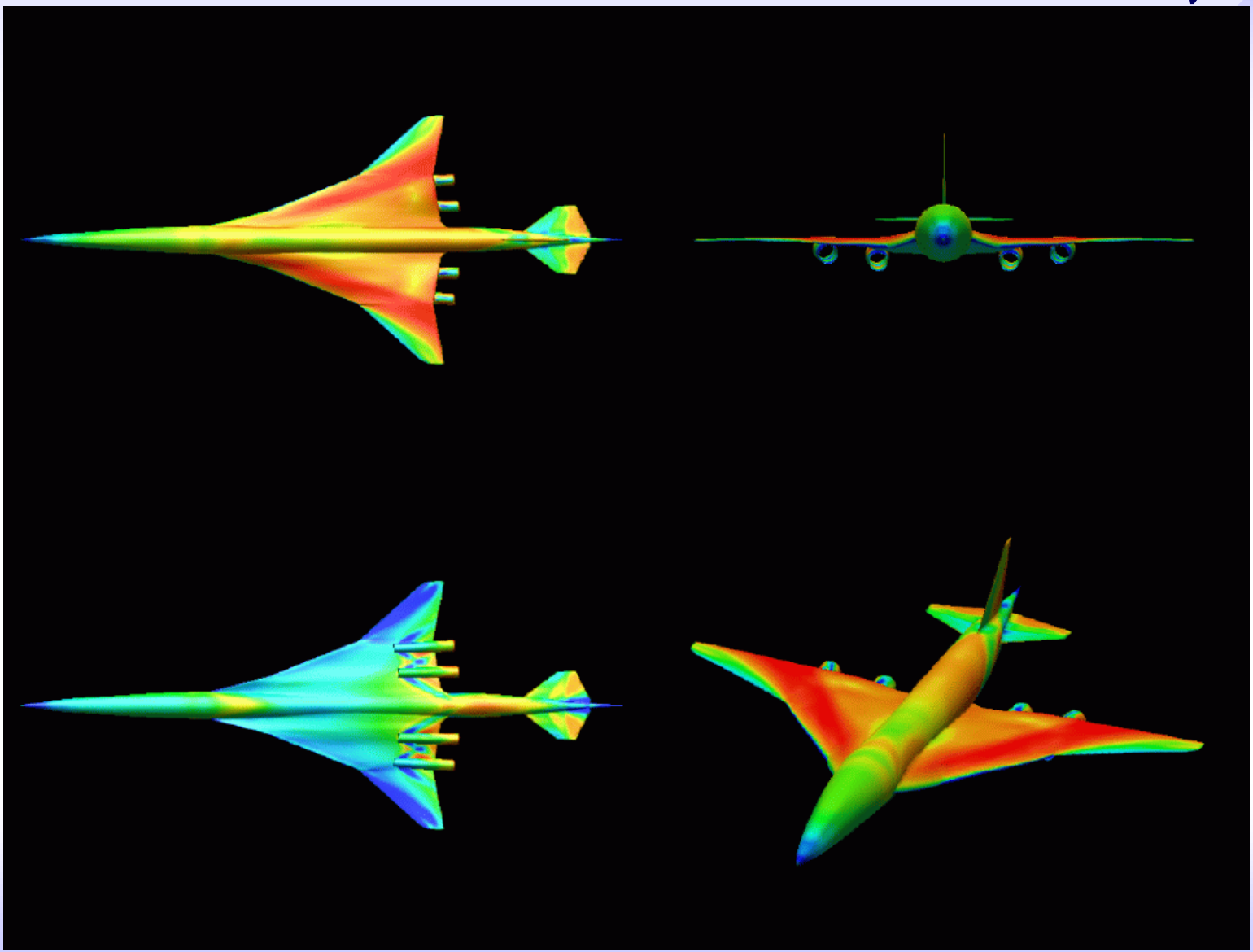
- *Sonic boom on ground has significant lateral influence (typically 25 miles for small aircraft at 30,000 ft)*
- *Real atmosphere effects create a region of silence earlier than uniform atmosphere results because of ray refraction due to changes of density with altitude*

Other Proposed Concepts



- Large number of concepts proposed, but none of them has gone through a detailed optimization process even with simplified linear techniques
- Unfortunately, for sonic boom, the devil is in the details…

Ames HSCT - AIRPLANE

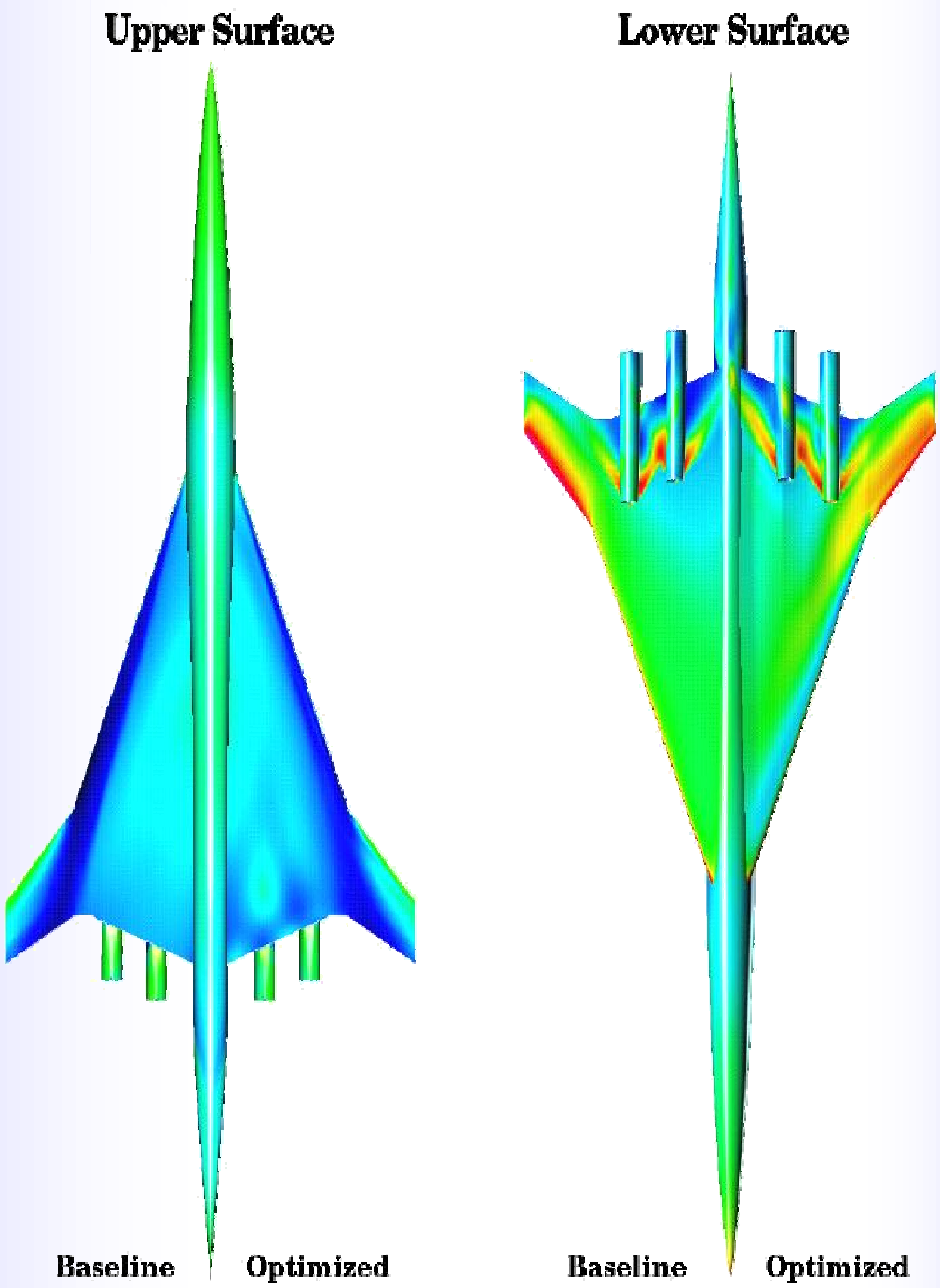


Ames HSCT - SYN107-MB

Supersonic Transport Configuration

Iso-Cp Contours

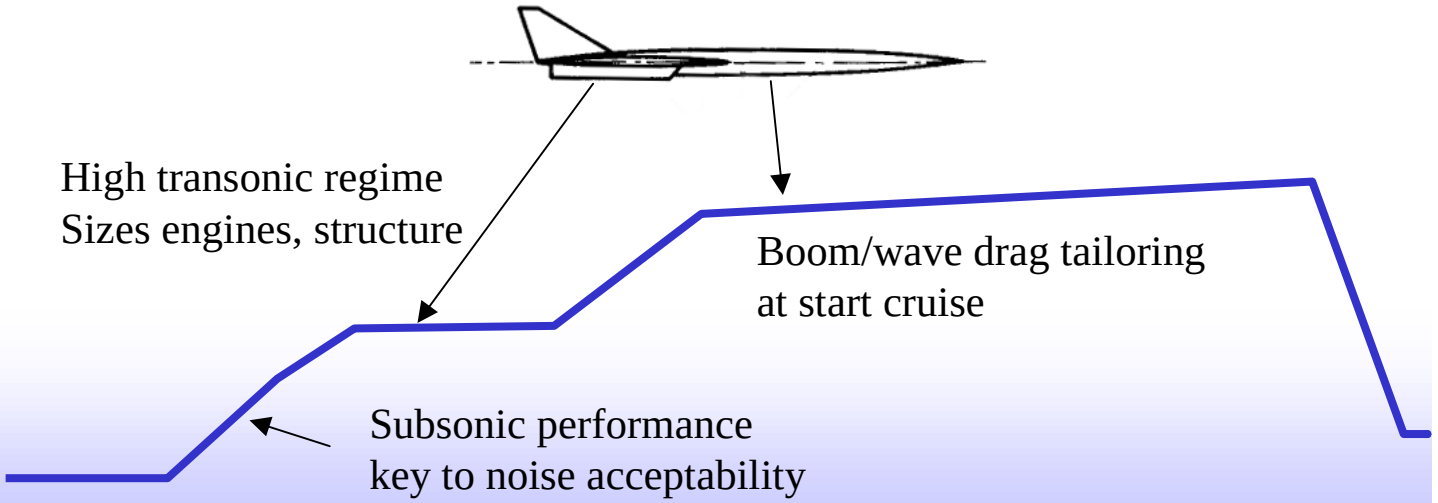
$M = 2.2, C_L = 0.105$



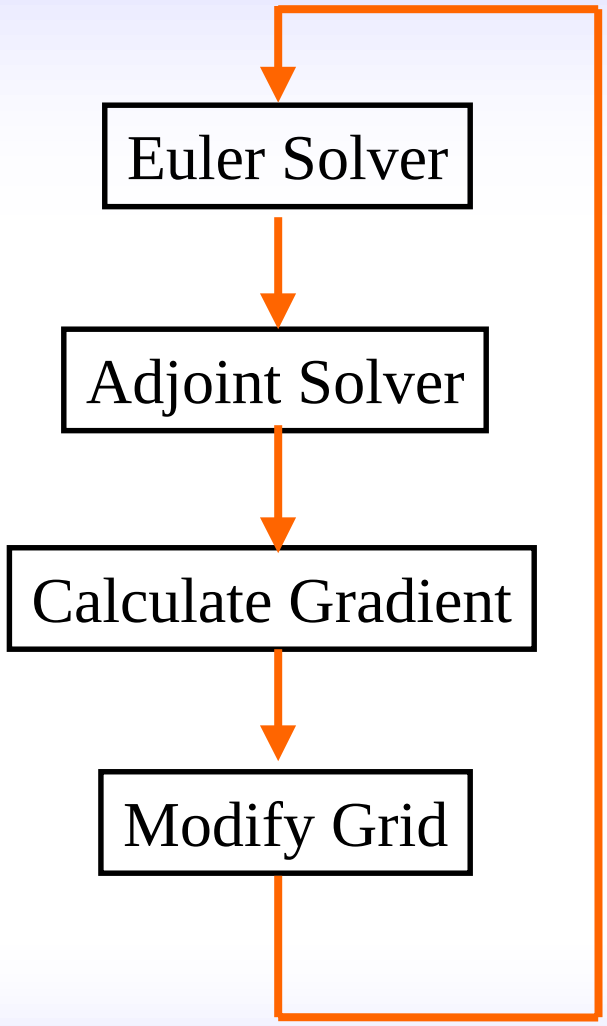
Advanced Algorithms for QSP Design

- **MDO: Additional disciplines of critical importance**
  - *Propulsion*
  - *Structure/Weights*
  - *Mission/Trajectory*

Multi-point optimization includes boom, cruise performance, off-design constraints



Two Dimensional Sonic Boom Minimization using the Adjoint Method



- **FLO83 flow solver**
  - Modified Runge-Kutta time stepping scheme
  - Jameson-Schmidt-Turkel (JST) scheme for artificial dissipation
  - Local time stepping, implicit residual smoothing, multigrid
- **Adjoint method**
  - Development of adjoint method for calculation of non-collocated sensitivities
- **Cost function**
  - Norm of the pressure difference between current and target pressure distributions

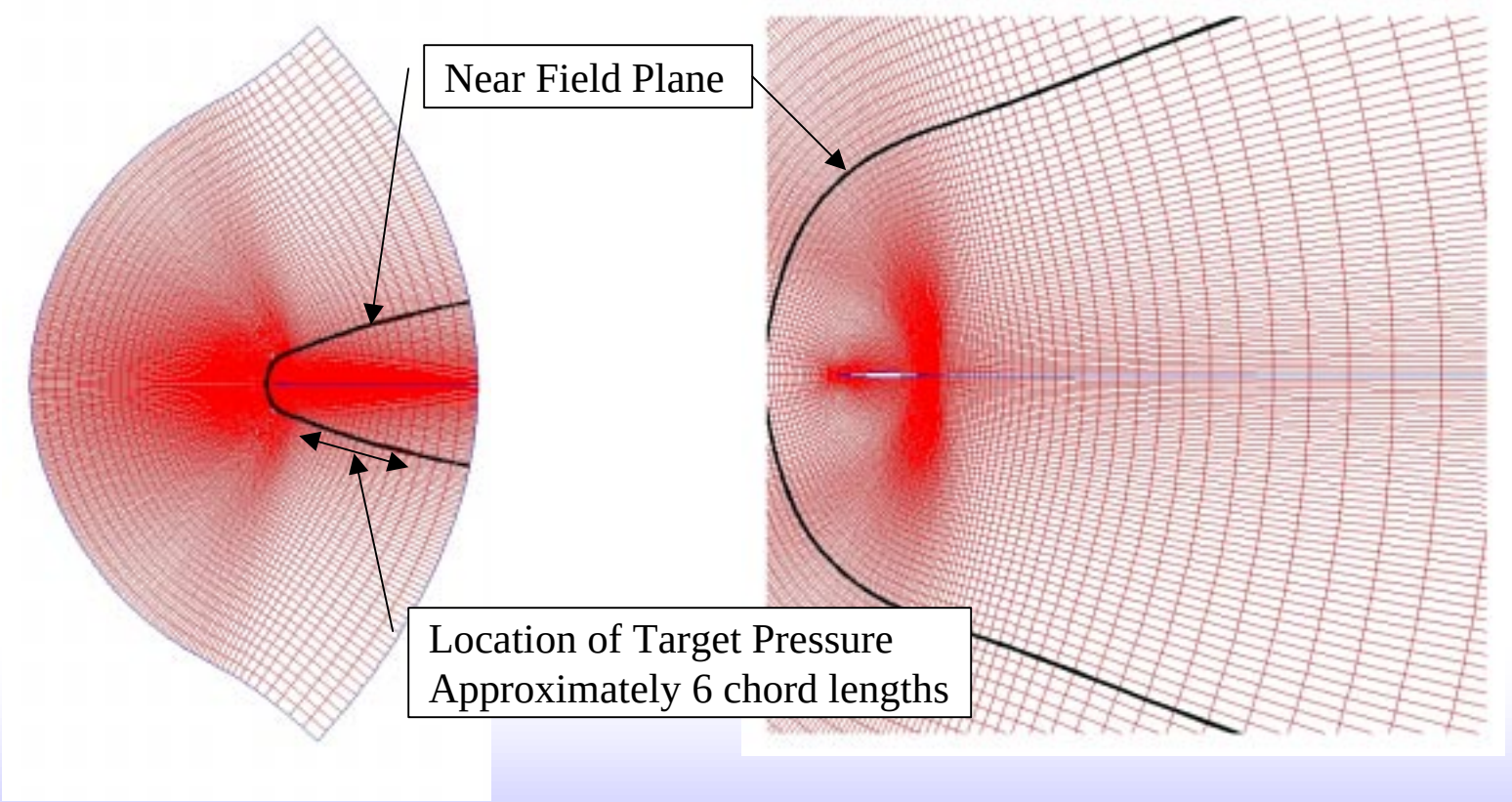
Two Dimensional Sonic Boom Minimization using the Adjoint Method

- **Flow Condition - Mach Number = 1.8**
- **Test Cases**
  - » **Case A** : Unique Near Field Target Pressure
    A bump is placed on the lower surface of the biconvex airfoil. The near field pressure distribution of the new airfoil is then used as the target pressure distribution for the design of the original airfoil

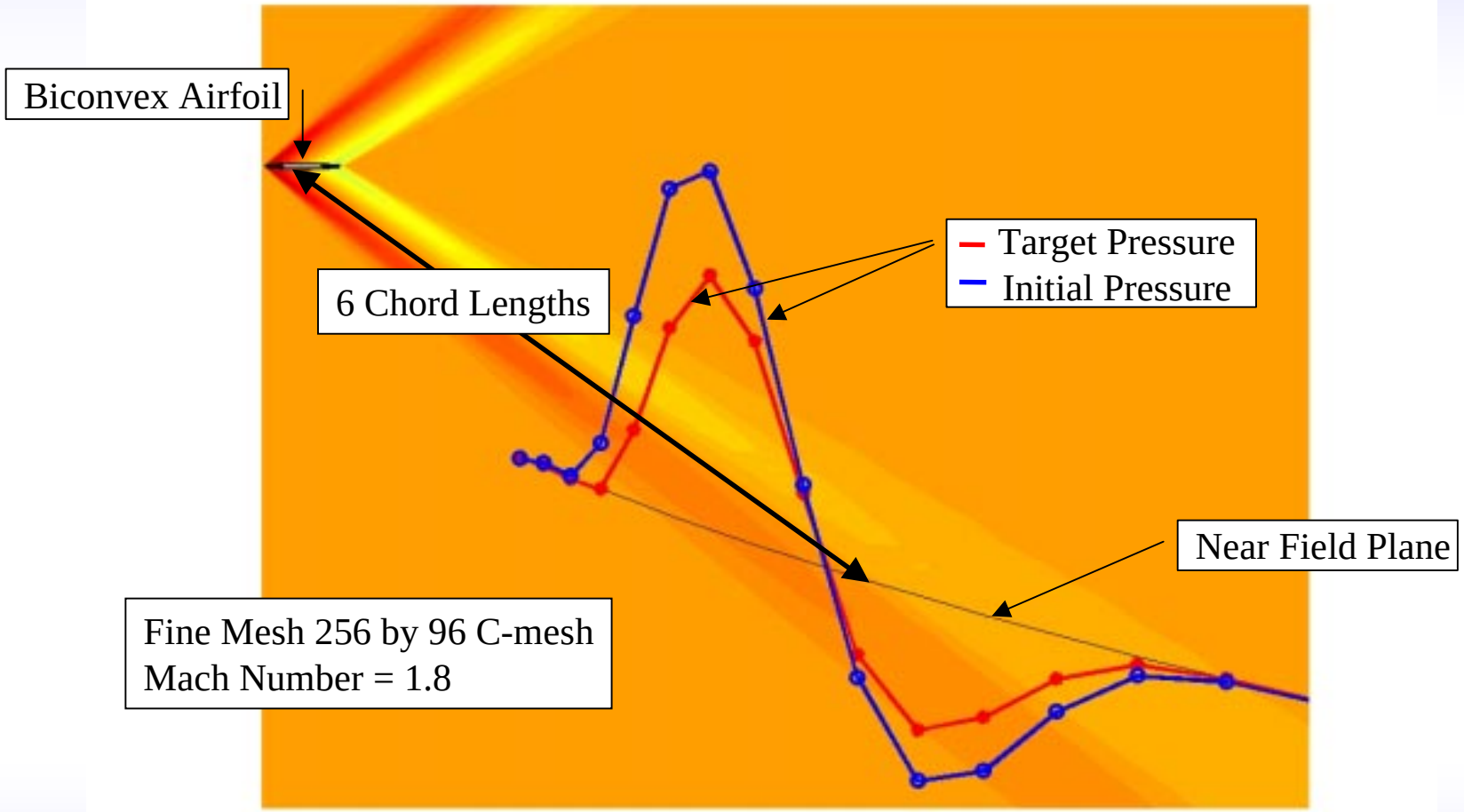
  - » **Case B** : Smoothed Near Field Target Pressure
    Near field target pressure obtained by implicitly smoothing the existing pressure.

Two Dimensional Sonic Boom Minimization using the Adjoint Method

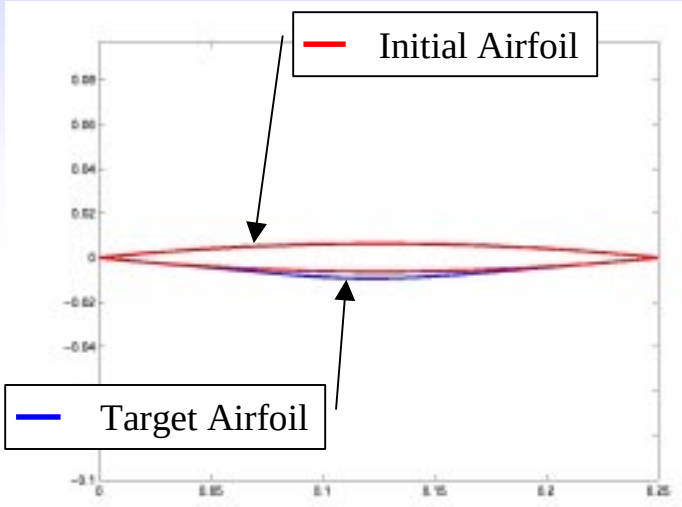
- **Airfoil Geometry and Mesh Size**
  - » Biconvex Airfoil with 5% Thickness Ratio.
  - » Fine Mesh Size - 256 by 96 C-mesh



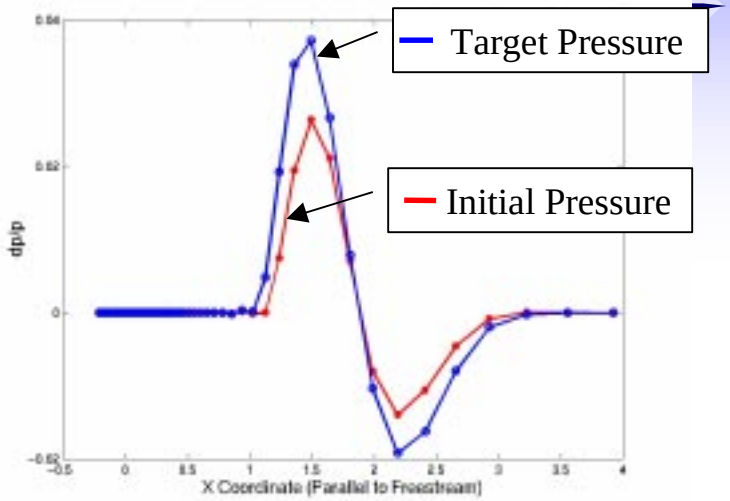
Geometry and Near Field Plane Description



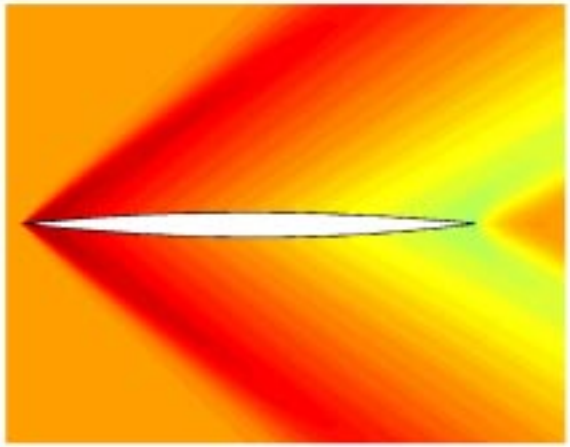
Initial Near Field Pressure Contours and Airfoil Geometry - (Case A, Mach Number = 1.8, Biconvex Airfoil)



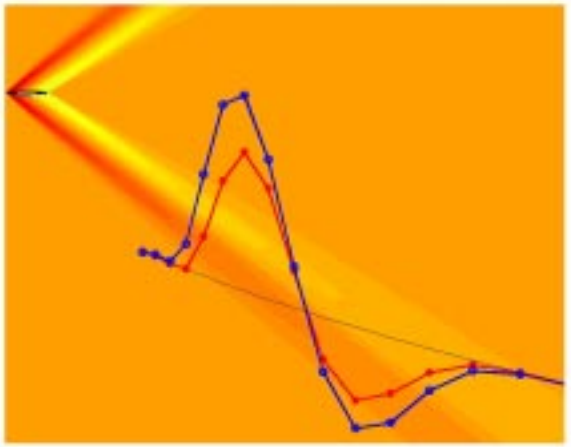
Initial and Target Airfoil Geometry



Initial and Target Near Field Pressure Distribution

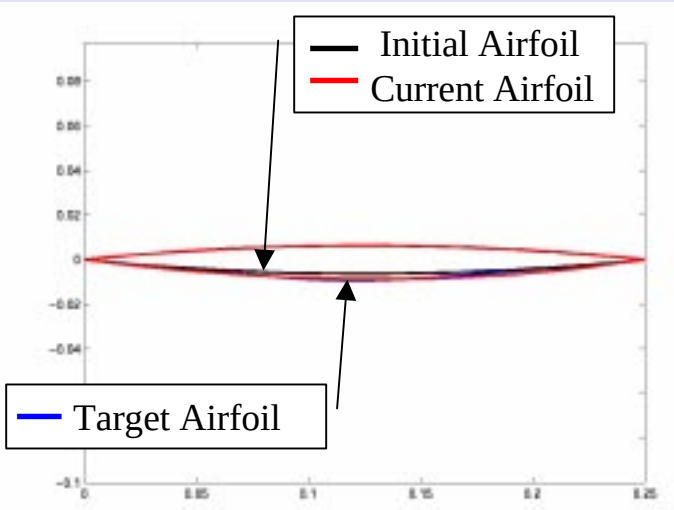


Pressure Contour Plot

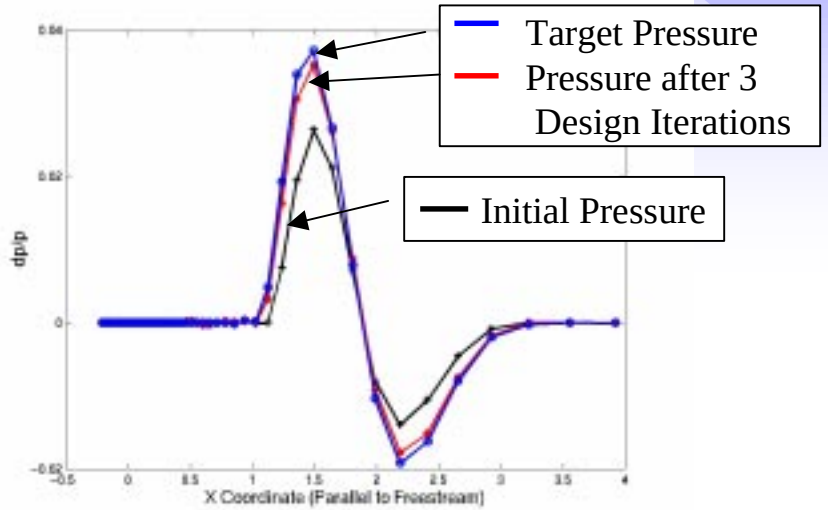


Near Field Pressure Contour Plot

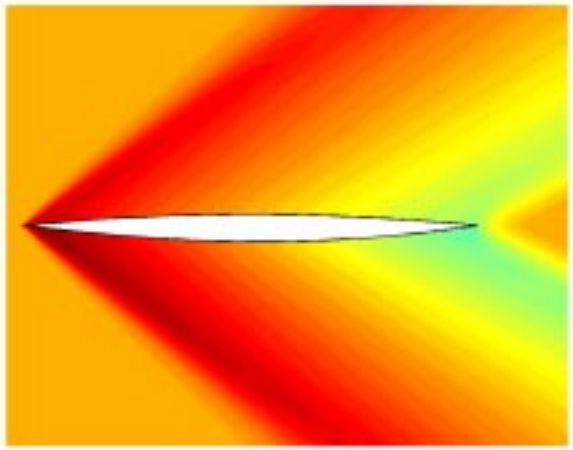
Pressure Contours and Airfoil Geometry After 3 Design Iterations - *(Case A, Mach Number = 1.8, Biconvex Airfoil)*



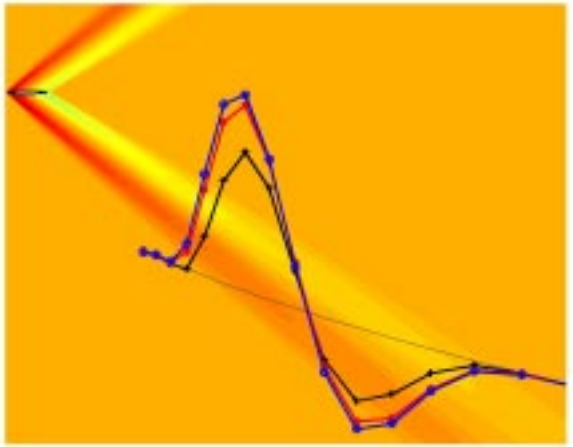
Initial, Current, and Target Airfoil Geometry



Initial, Current, and Target Near Field Pressure Distribution

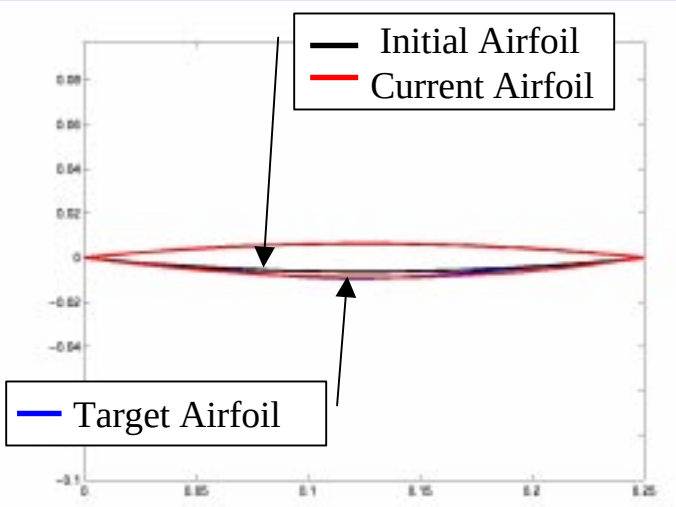


Pressure Contour Plot

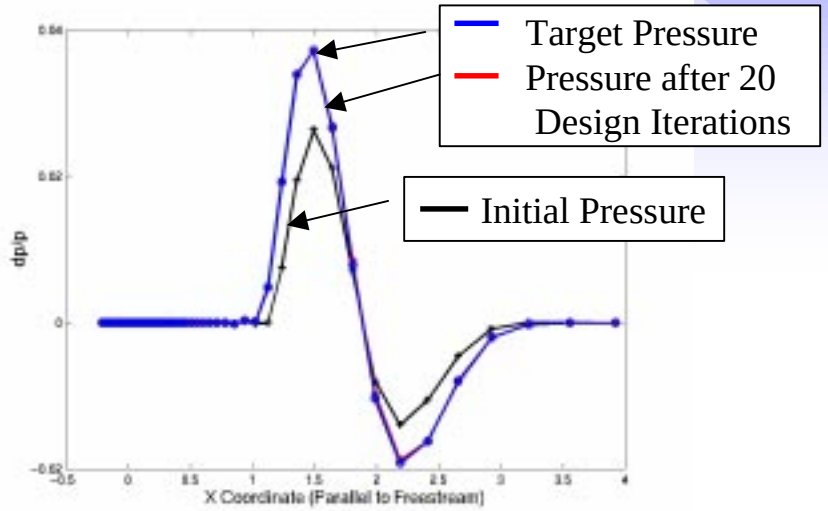


Near Field Pressure Contour Plot

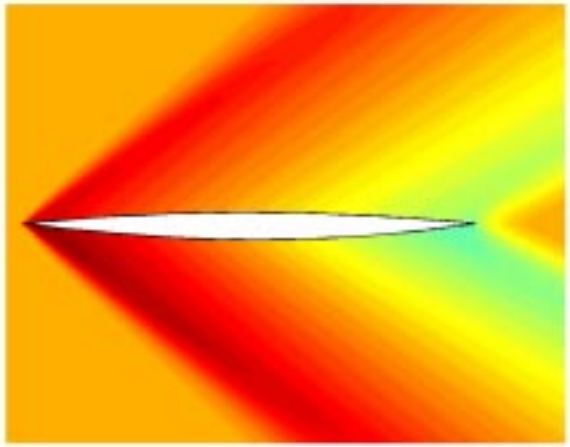
Pressure Contours and Airfoil Geometry After 20 Design Iterations - (Case A, Mach Number = 1.8, Biconvex Airfoil)



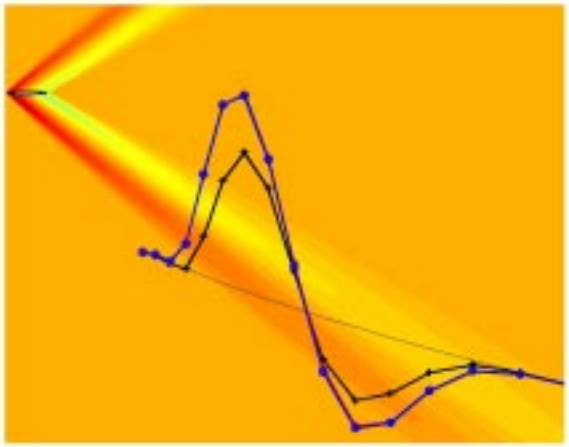
Initial, Current, and Target Airfoil Geometry



Initial, Current, and Target Near Field Pressure Distribution

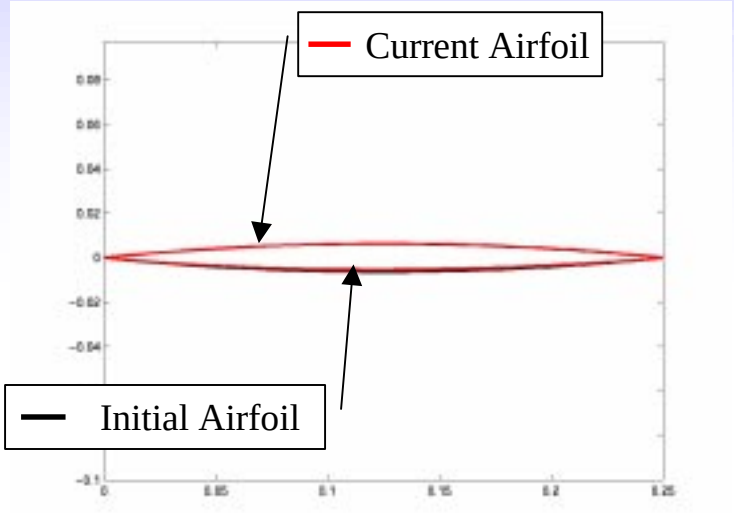


Pressure Contour Plot

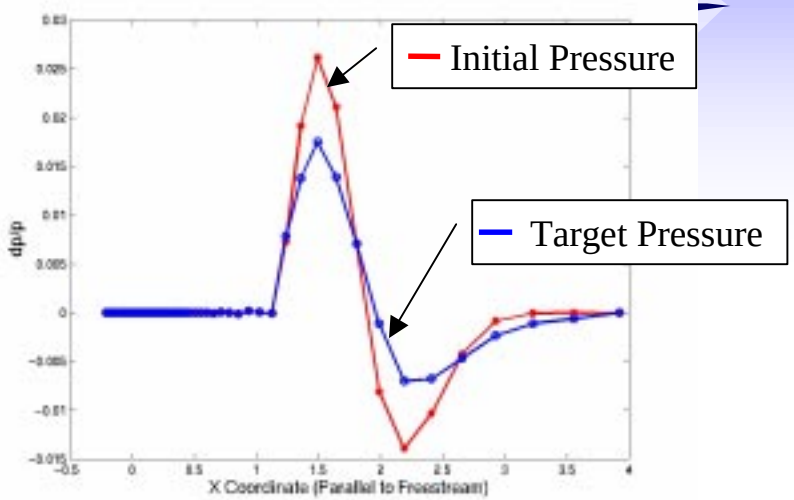


Near Field Pressure Contour Plot

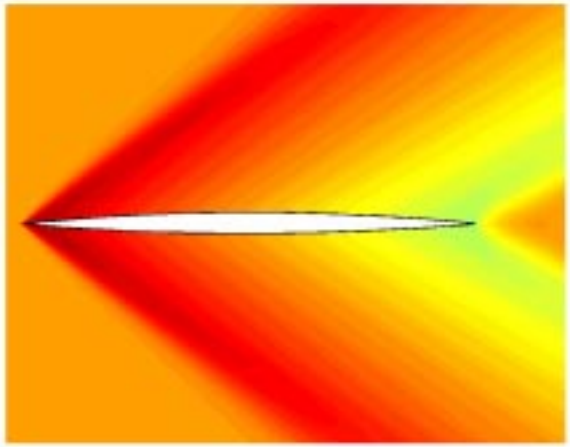
Initial Near Field Pressure Contours and Airfoil Geometry - (Case B, Mach Number = 1.8, Biconvex Airfoil)



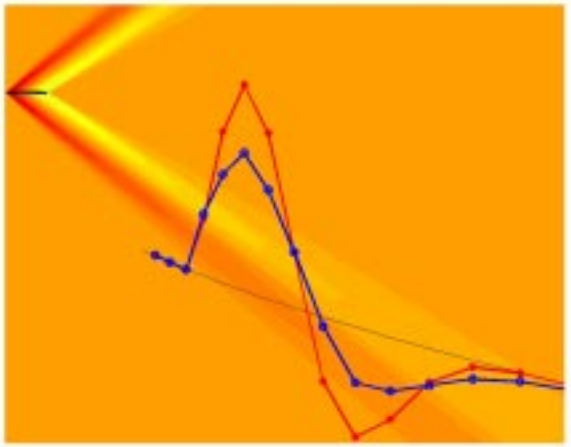
Initial and Target Airfoil Geometry



Initial and Target Near Field Pressure Distribution

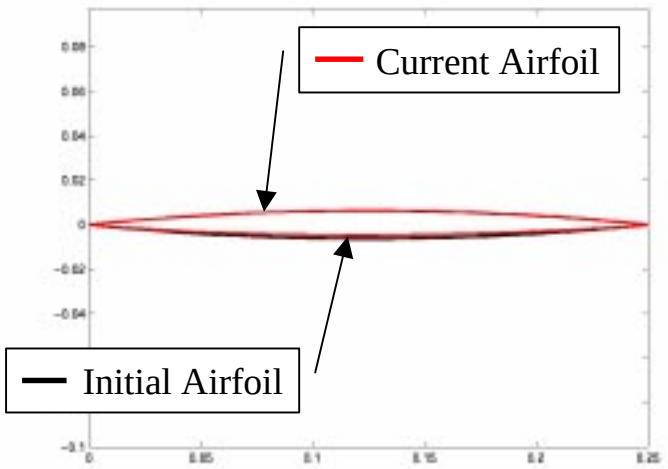


Pressure Contour Plot

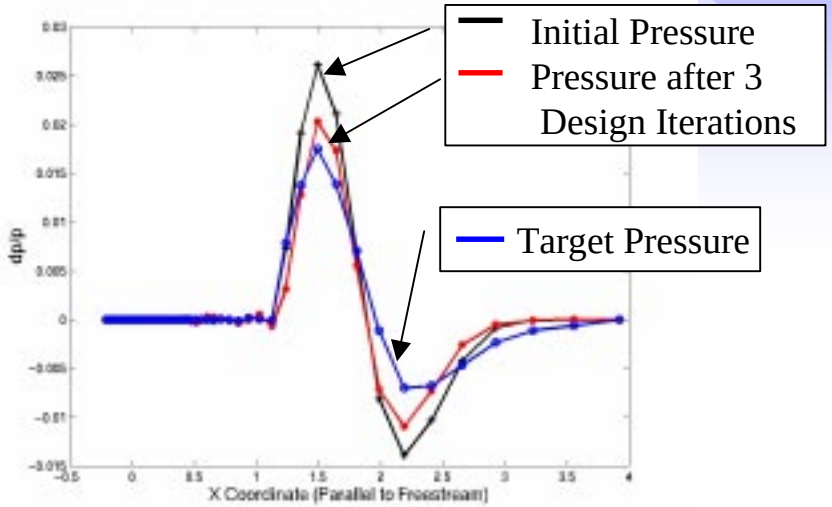


Near Field Pressure Contour Plot

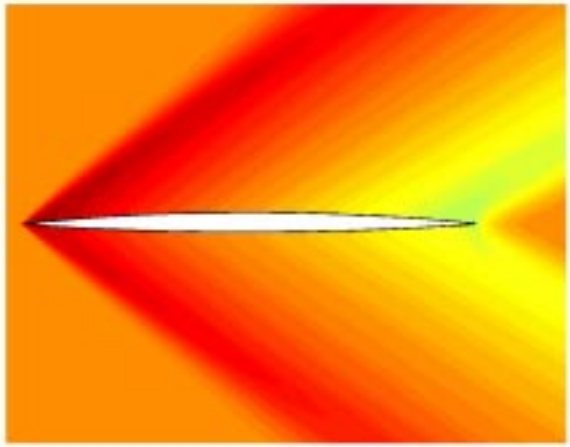
Pressure Contours and Airfoil Geometry After 3 Design Iterations - (Case B, Mach Number = 1.8, Biconvex Airfoil)



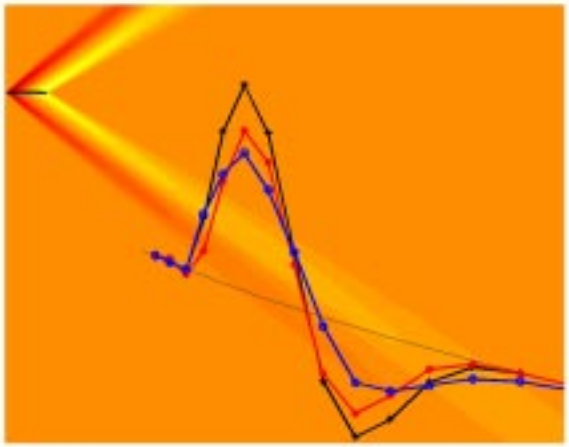
Initial, Current, and Target Airfoil Geometry



Initial, Current, and Target Near Field Pressure Distribution

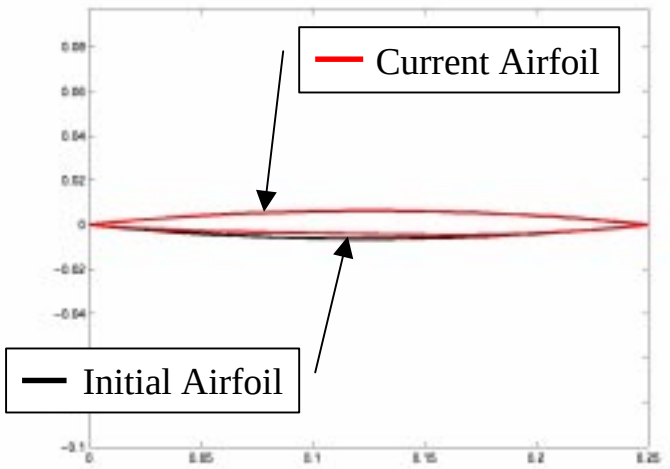


Pressure Contour Plot

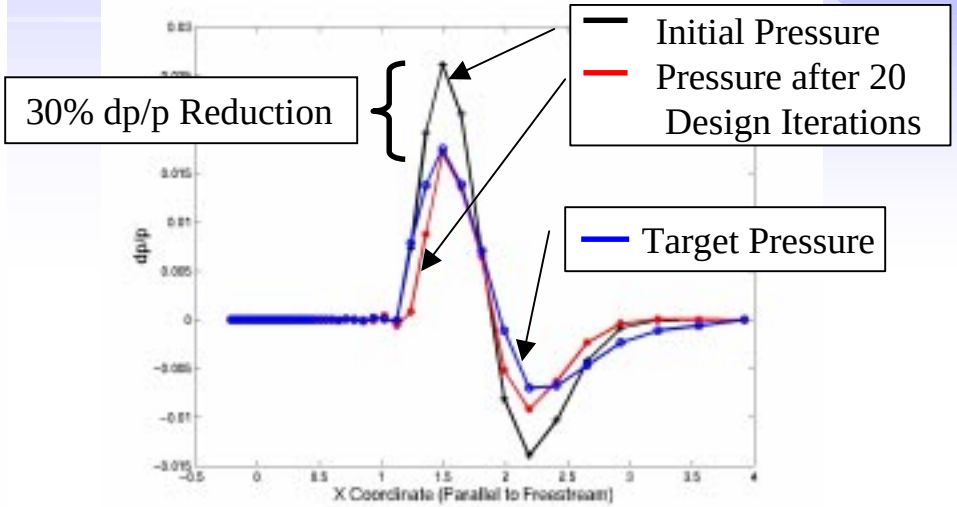


Near Field Pressure Contour Plot

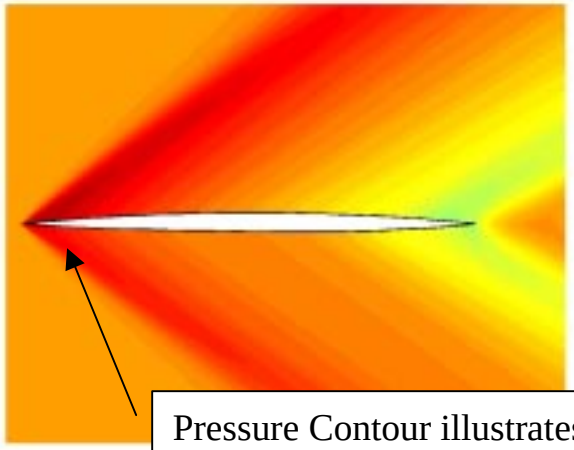
Pressure Contours and Airfoil Geometry After 20 Design Iterations - (Case B, Mach Number = 1.8, Biconvex Airfoil)



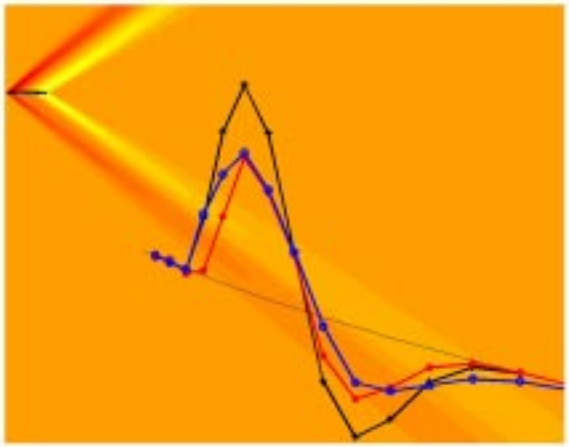
Initial, Current, and Target Airfoil Geometry



Initial, Current, and Target Near Field Pressure Distribution



Pressure Contour illustrates the Reduction in Shock Strength on Lower Surface



Near Field Pressure Contour Plot

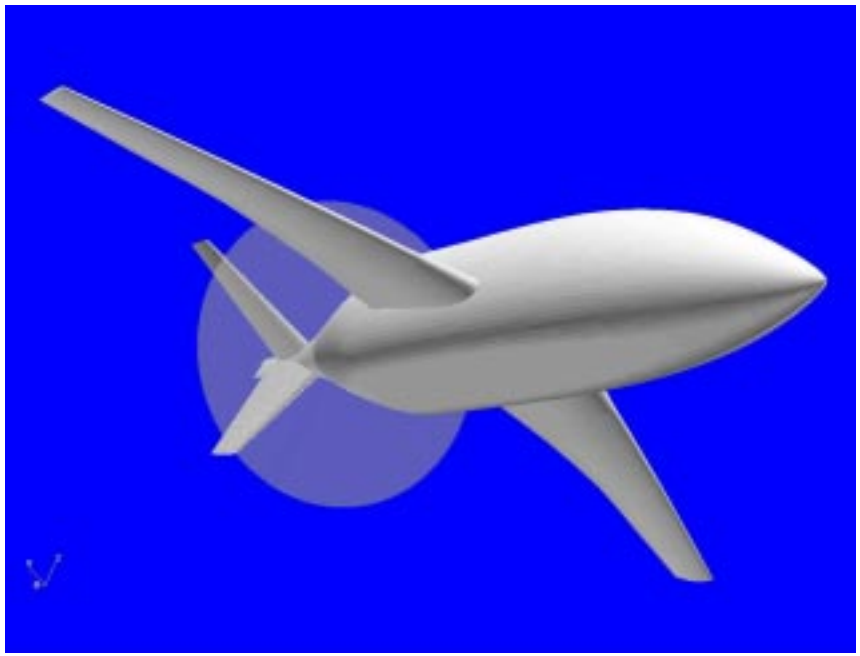
Application of Viscous Adjoint Design

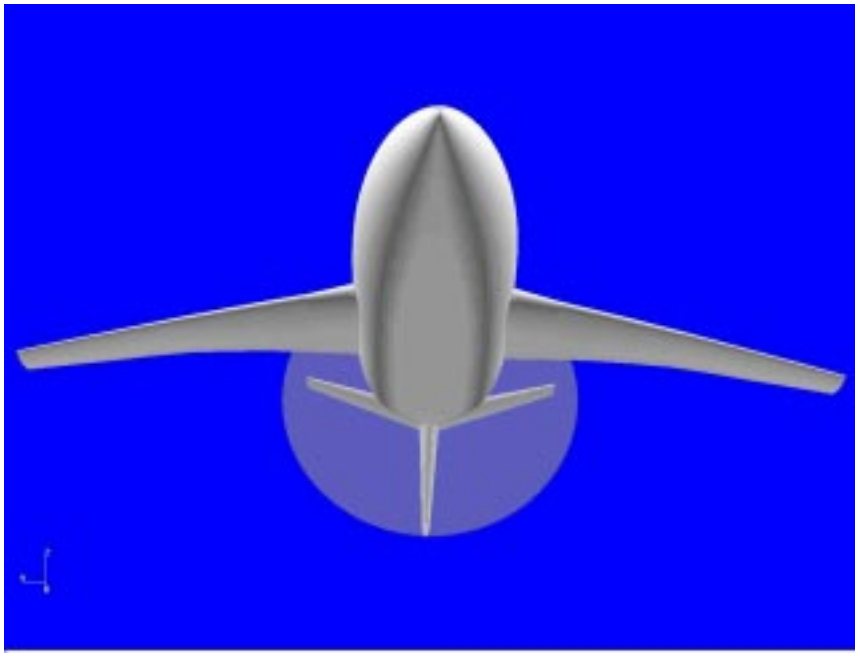
- *Proposed design for a Reno Air Racer*
- *Constraint: piston engine + propeller*
- *Currently won by WWII aircraft (Bearcat, P51, Hawker SeaFury extensively modified)*
- *Typical top speed = 500 mph*

Application of Viscous Adjoint Design

- *Current Design: 550mph*
- *“Cruise” Mach number = 0.78*
- *Large airfoil thickness and low sweep*
- *C_L = 0.27 (for a 3g turn)*
- *Aero design carried out by small team*

3D Views

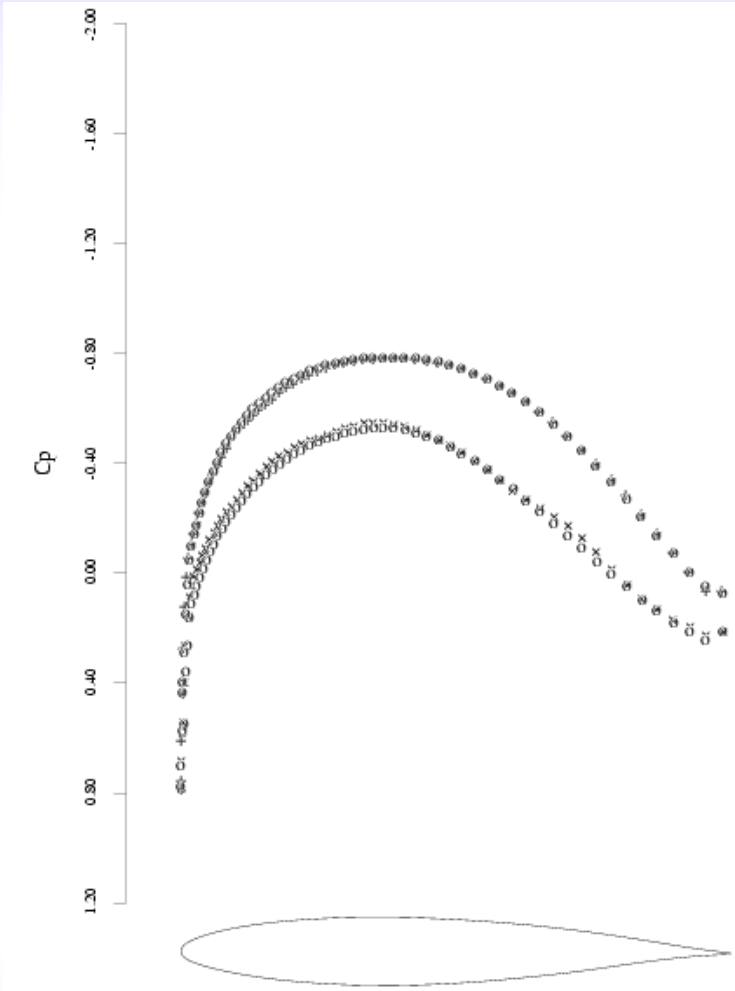


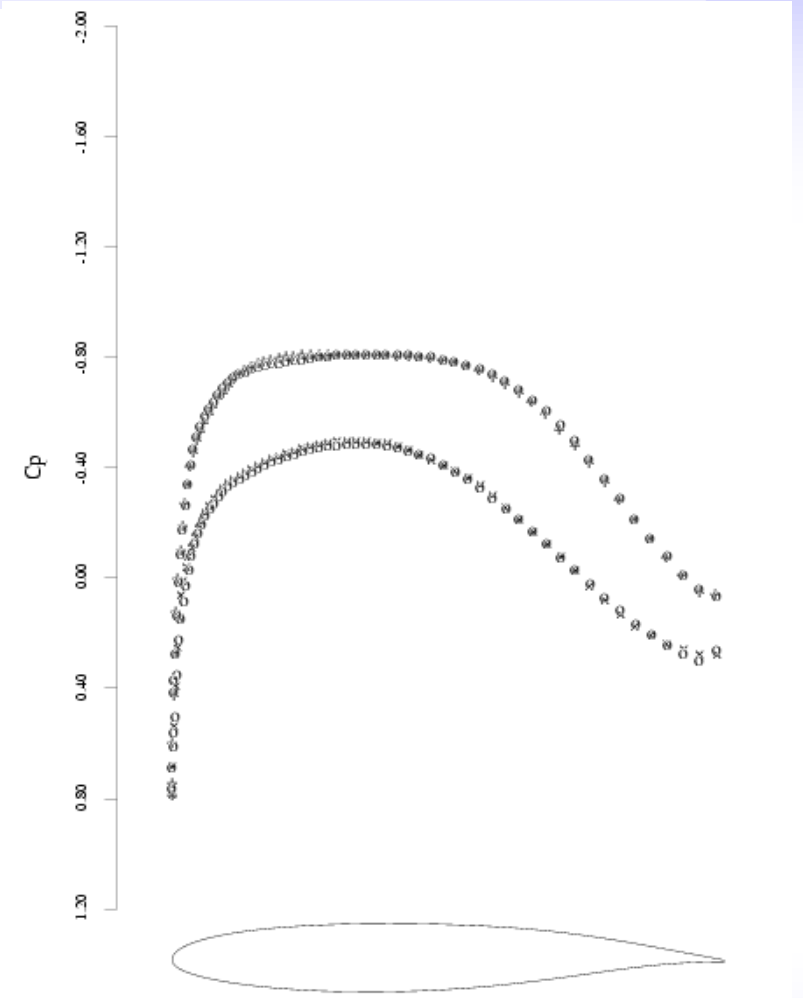


Design Procedure

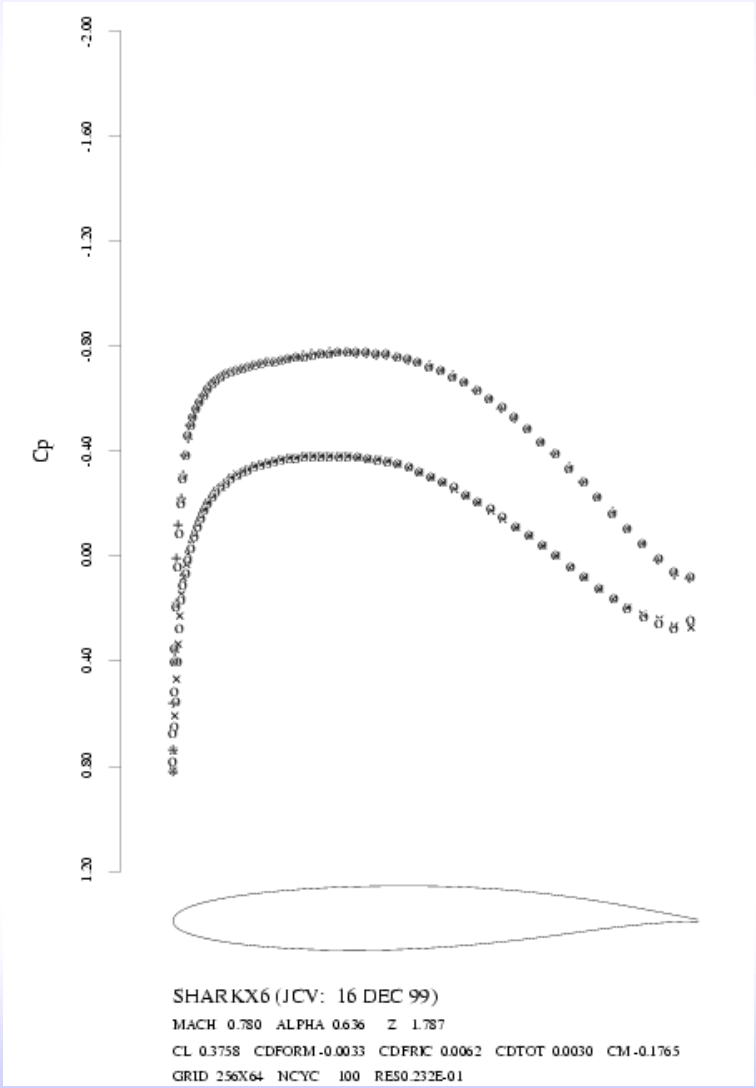
- ***Initial multipoint Euler design (at a variety of C_Ls)***
  - *Strong normal shock in wing inboard region indicated a need to modify the fuselage-wing intersection*
  - *Extended fuselage eliminated problems in inboard wing*
- ***Single point full viscous design using the full NS adjoint***
- ***Pressure distribution cleanup using NS inverse design***
  - *Favorable pressure gradient to promote laminar flow run*
- ***Result: Shock-free pressure distributions with very good off-design behavior***

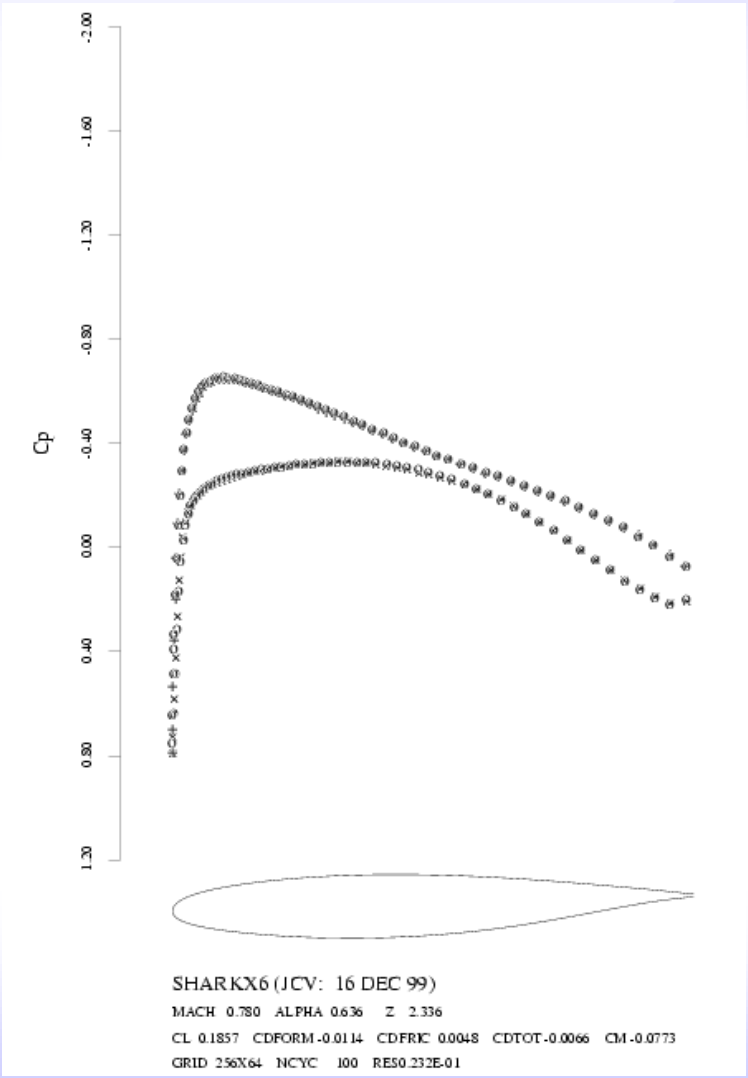
Resulting Pressure Distributions



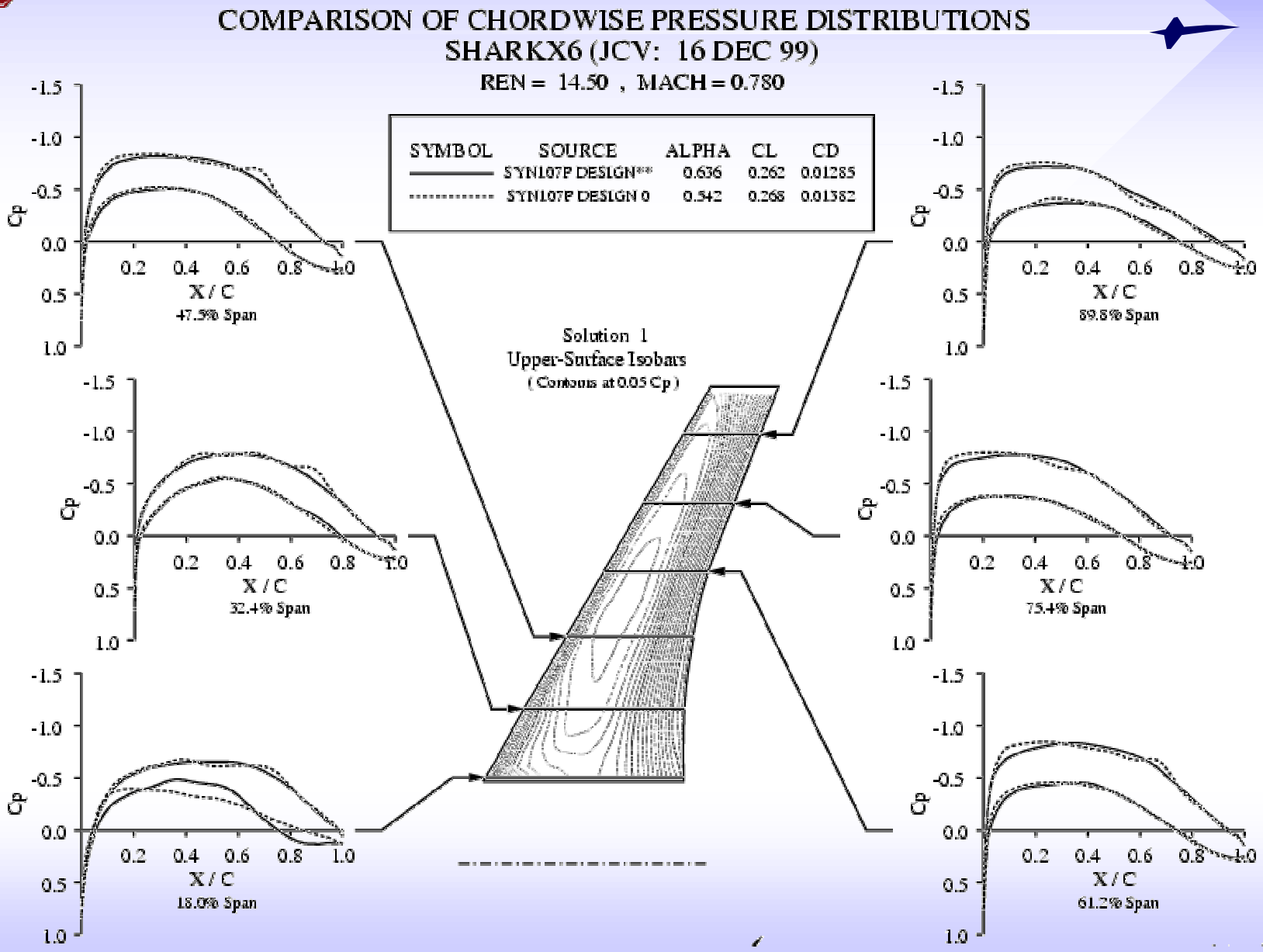


Resulting Pressure Distributions

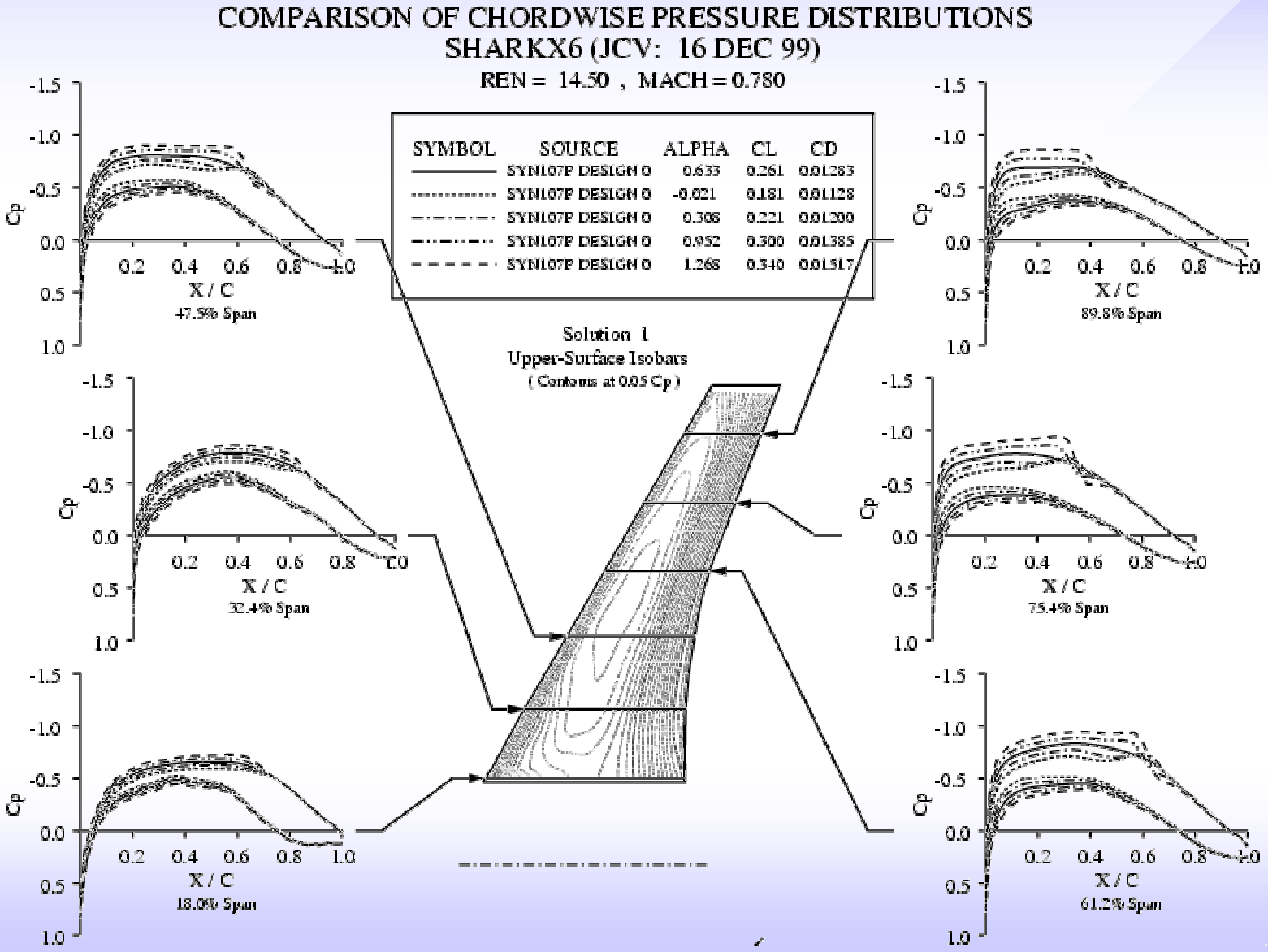




Resulting Pressure Distributions



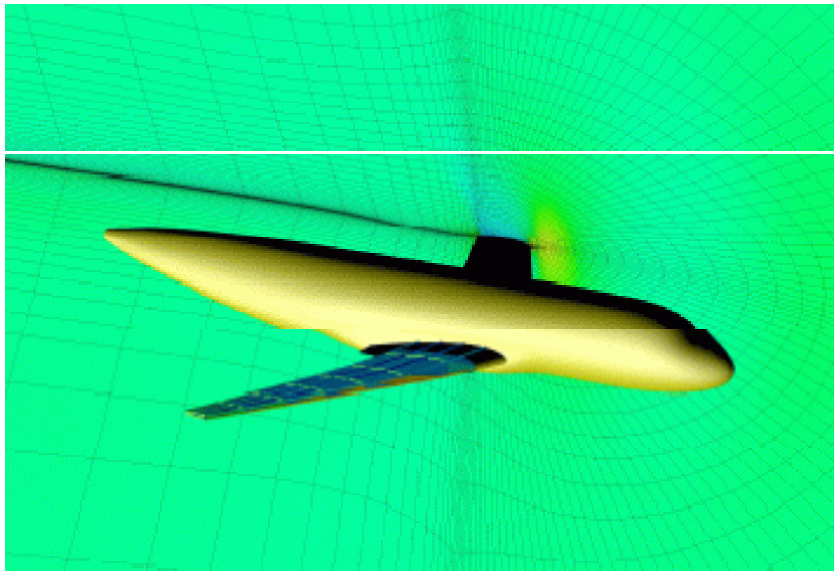
Resulting Pressure Distributions



Complex Configuration Design

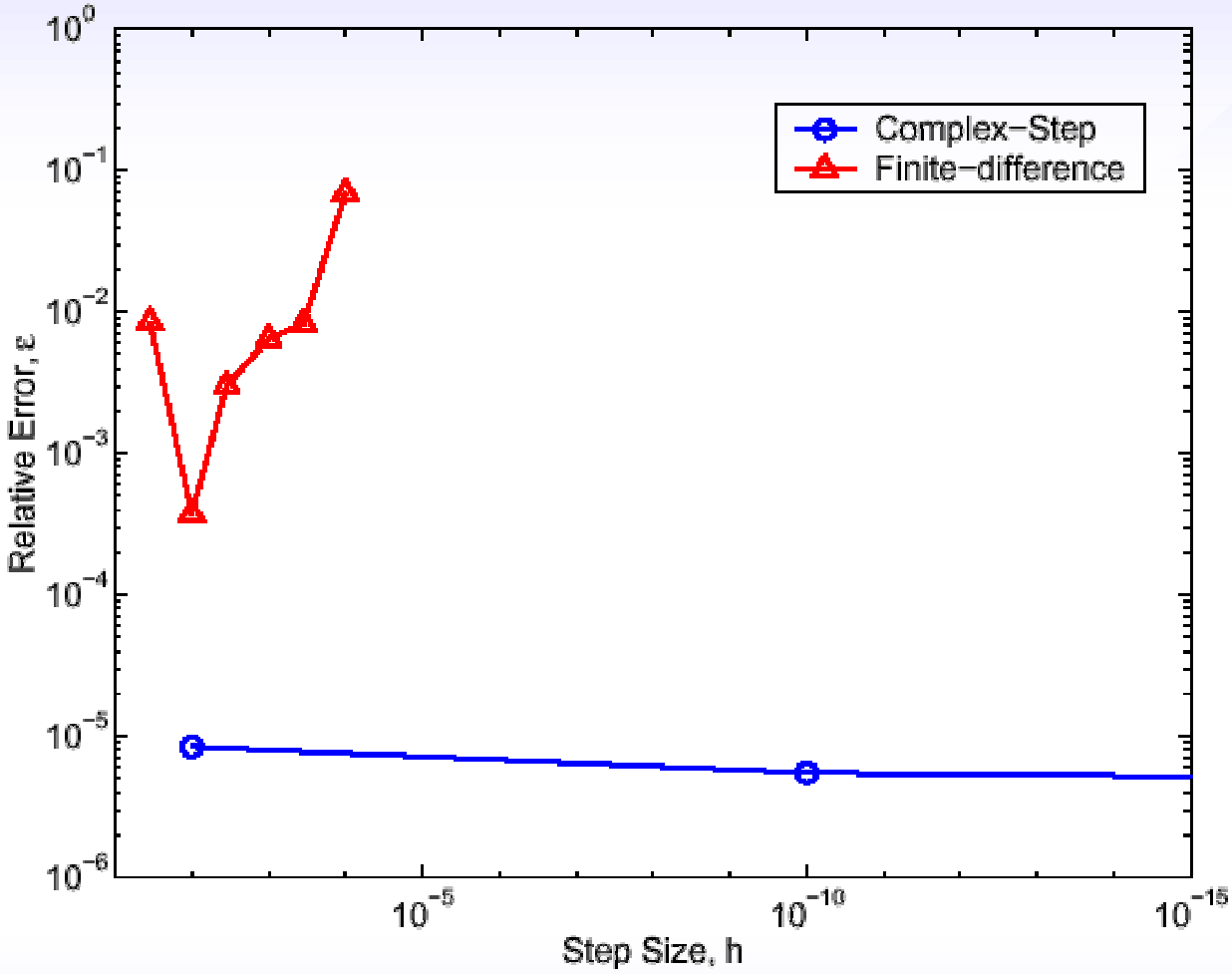
- ***Multiblock version of adjoint-based NS design exists***
- ***Extensive development required for an industrial strength design method***
  - *Rewrite of code is already under way*
- ***Ability to redesign arbitrarily complex configurations including wing, fuselage, pylons, nacelles, diverters (SST)***
- ***Grid generation is the major bottleneck***
- ***Parallel implementation is absolutely necessary to reduce wall-clock time***

Complex-Step Method For Aero-Structural Design



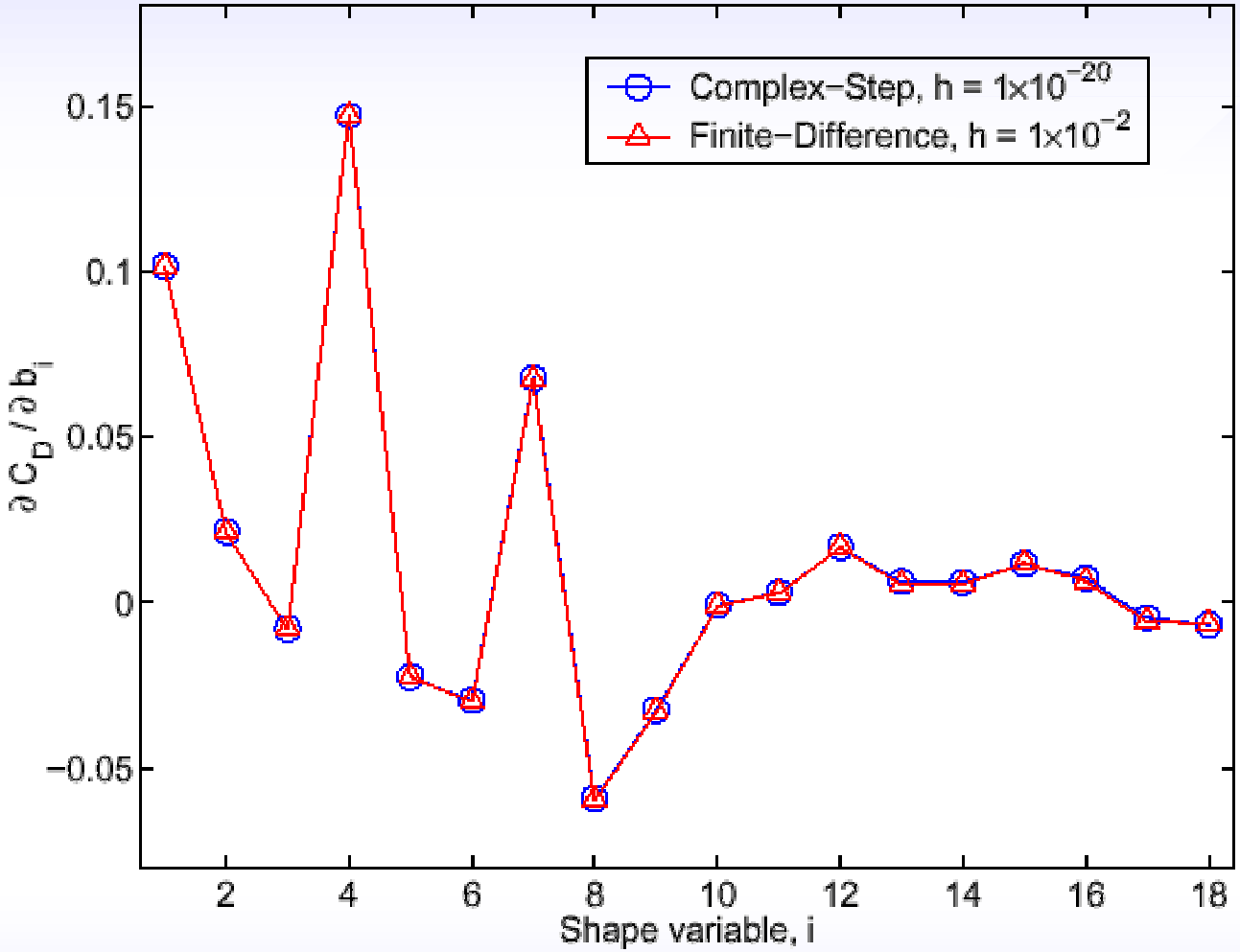
- Aerodynamics: SYN107-MB, a parallel, multiblock Navier-Stokes flow solver.
- Structures: detailed finite element model with plates and trusses.
- Coupling: high-fidelity, consistent and conservative.
- Geometry: centralized database for exchanges (jig shape, pressure distributions, displacements.)
- Coupled-adjoint sensitivity analysis

Complex-Step Method Accuracy



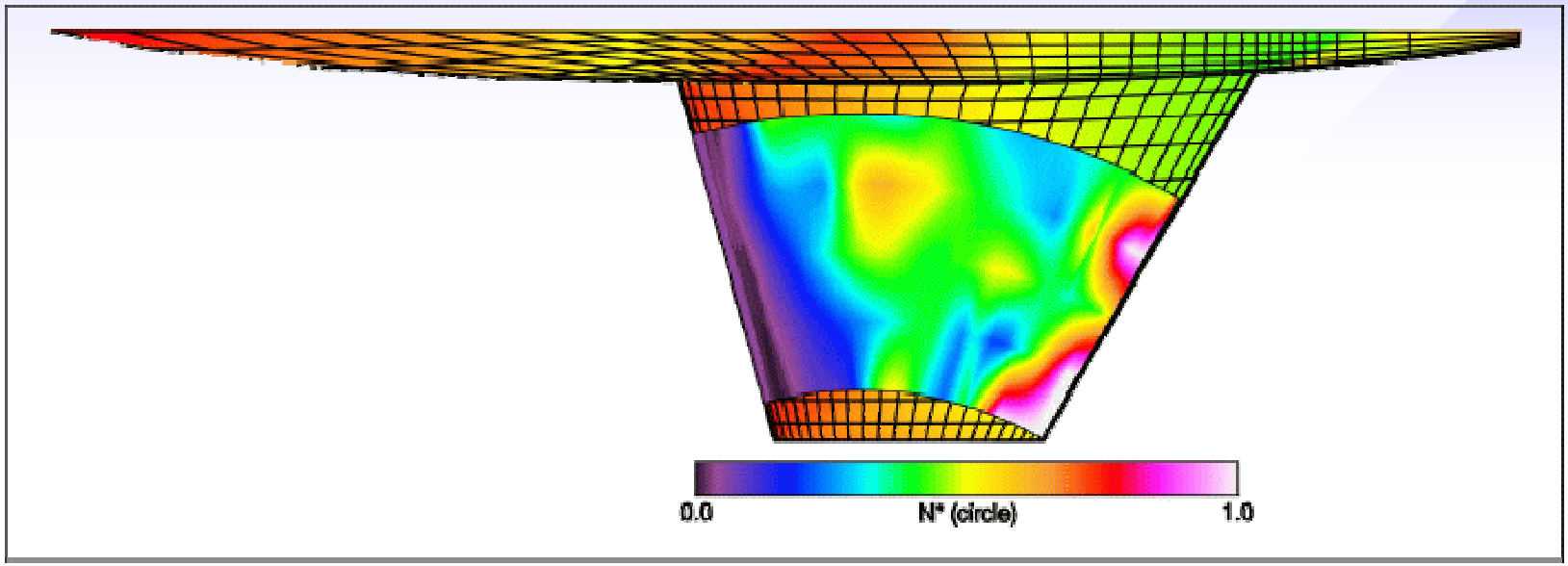
Finite-difference lacks robustness.

Complex-Step Method
Full Gradient



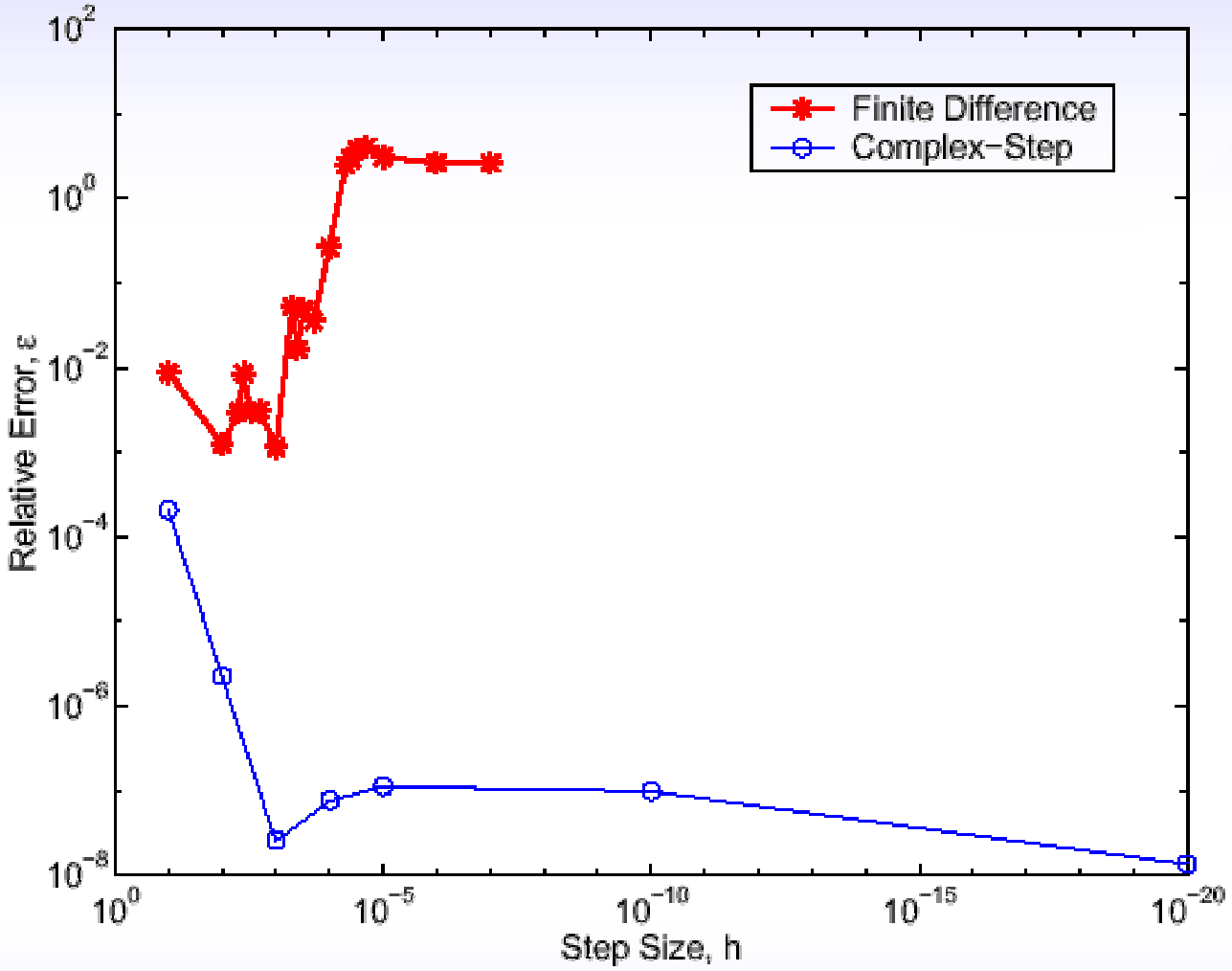
Much effort expended in choosing a finite-difference step.

Complex-Step Method
Natural Laminar Flow Aircraft



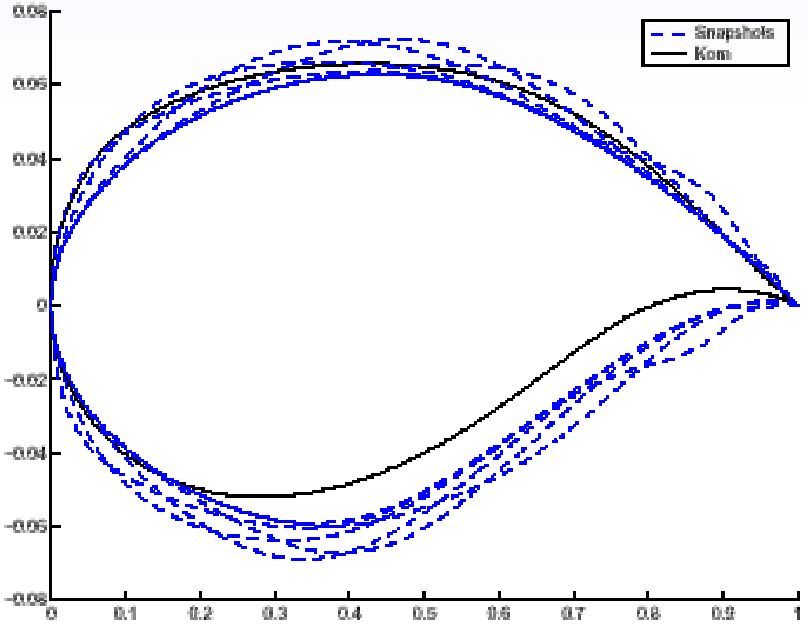
- Transition prediction
- Viscous and inviscid drag
- Design optimization:
  - Wing planform and airfoil design
  - Wing-Body intersection design
- Three programming languages: Fortran, C++ and Python.

Complex-Step Method

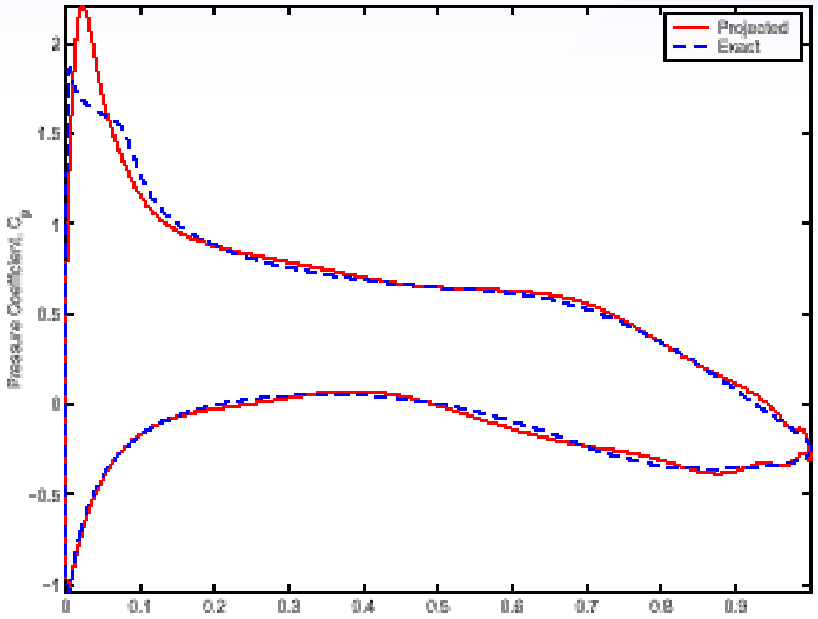


Finite-difference only useful for well chosen h.

Proper Orthogonal Decomposition

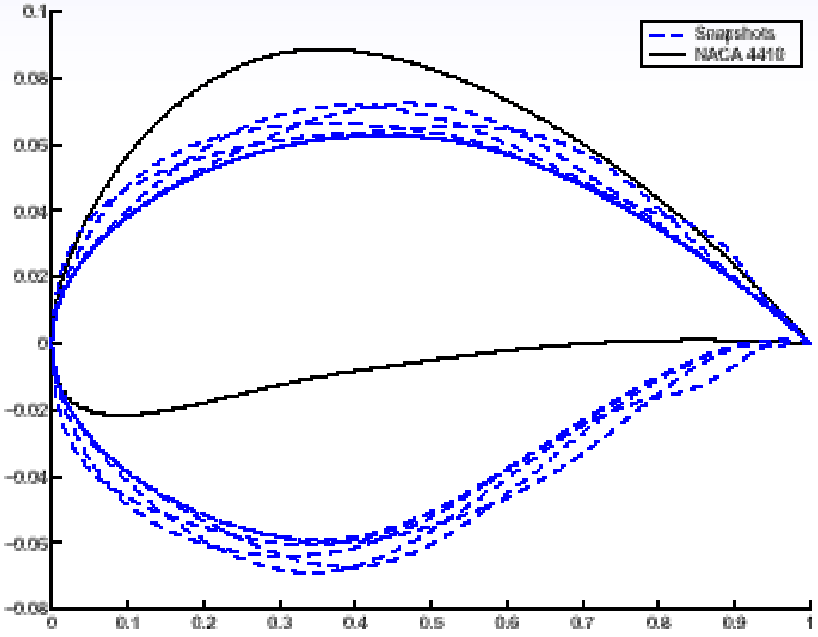


(a) Snapshots and Korn Airfoil

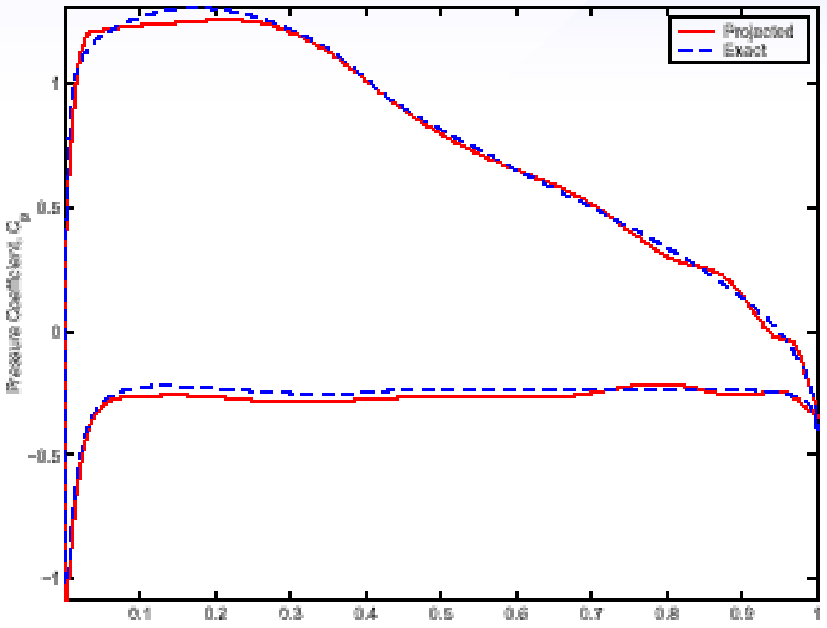


(b) C_p Comparison for Korn Airfoil

Proper Orthogonal Decomposition



(a) Snapshots and NACA 4410 Airfoil



(b) C_p Comparison for NACA 4410 Airfoil

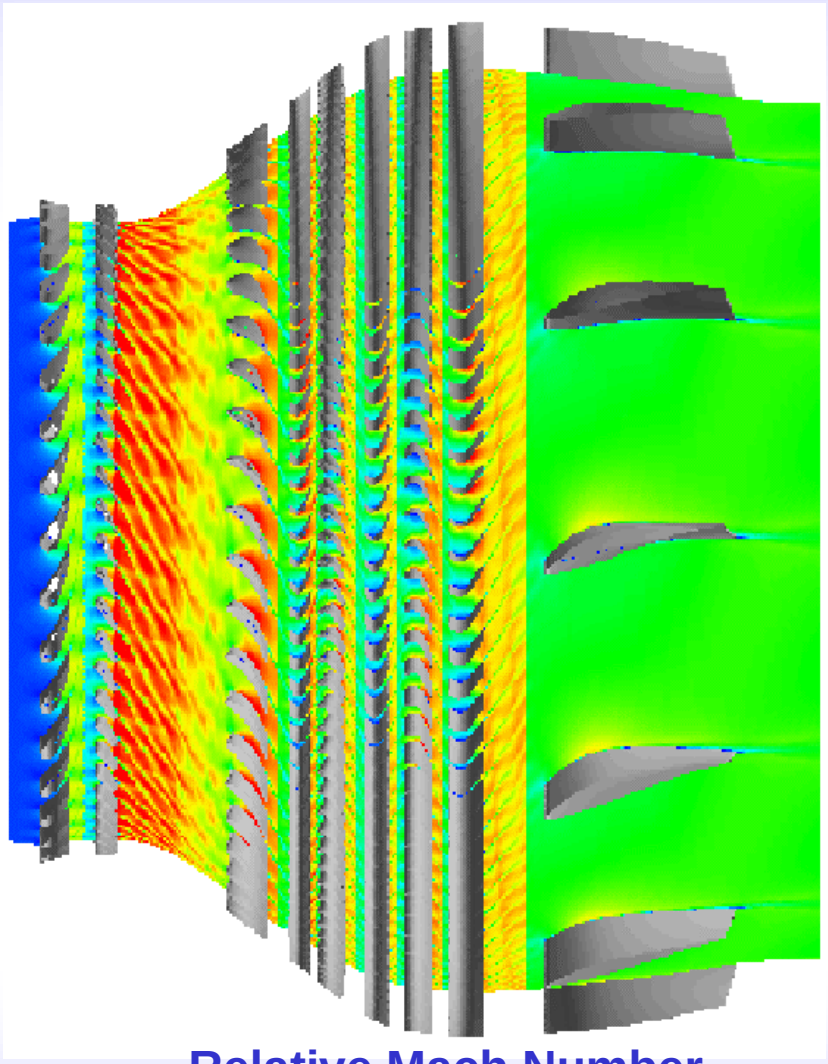
Conclusions

- ***There are many methods to compute sensitivities to systems governed by the Euler and Navier-Stokes equations, but the adjoint-method is the most computationally efficient by far***
- ***Other approaches can be used for small dimension design problems or to validate more sophisticated efforts***
- ***Much work remains to be done in the following areas***
  - *Coupled system sensitivity analysis (especially if the bandwidth of communication is large such as in aero-structural design)*
  - *Robustness and accuracy of sensitivity calculation procedures*
  - *Applications to innovative design concepts*

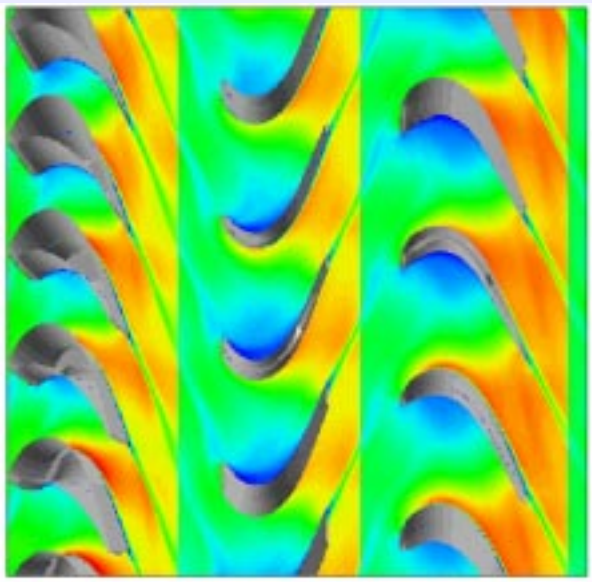
Current and Future Work

- *Low sonic boom aircraft*
- *MDO and aero-structural design*
- *Reduced order modeling*
- *Extensions to unsteady flows (aeroelastics and turbomachinery)*
- *POD for bandwidth reduction in complex multidisciplinary designs*

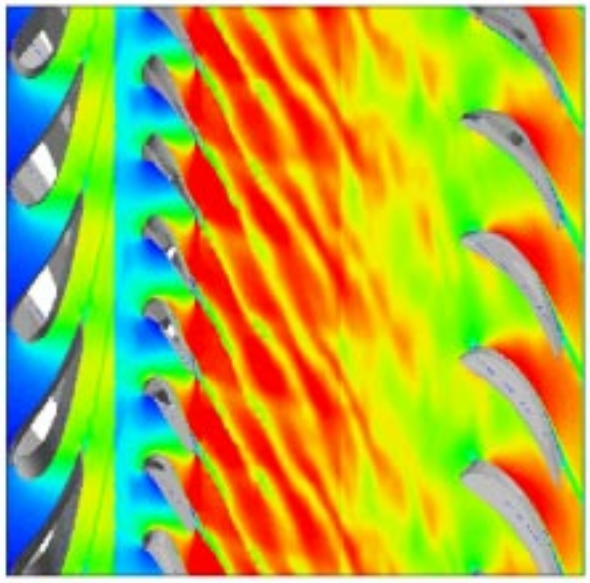
Unsteady Flow Simulation of PW6000 Turbine (TFLO)



Relative Mach Number



LPT (5,6,7)



HPT (1,2,3)